AIRSCREWS

AIRSCREWS
IN THEORY AND EXPERIMENT

BY

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PREFACE

AN endeavour has been made to present in this work an accurate and comprehensive account of the science of the airscrew from both its theoretical and experimental aspects.

Since the purpose of mechanical philosophy is to give a clear and ordered exposition of mechanical phenomena, it follows that the validity of a theory depends on the truth and comprehensiveness of the fundamental assumptions on which the theory is based. When the conception of the natural phenomena is imperfect, or when these phenomena are completely understood, but are so complex that the problem cannot be solved by mathematical methods, the aid of experiment is sought. With problems of such difficulty there is a tendency to evolve either a theory of which the dominant attribute is empiricism, or one of which the underlying conception is essentially of a hypothetical nature. From the standpoint of practical utility each of these two types of theory has its limitations. Empiricism in any theory usually implies some lack of generality and also a tacit admission of the failure to comprehend the essence of the problem. On the other hand a theory which does not take into consideration all the experimental facts may illustrate, in a general manner, the influence of the controlling factors on the characteristics of the problem.

Many theoretical and experimental investigations of the problems peculiar to the airscrew have already been made, and there would appear to be abundant possibilities of further exploration. There is, however, no comprehensive theory embodying all the experimental data which have been accumulated. Even if such a theory were available, it would probably be unduly cumbersome in application. Nevertheless, the principal airscrew theories often yield, when in experienced hands, surprisingly good results.

In the preparation of the book the author has sought information from all available sources, particularly from scientific papers, published both at home and abroad. The results of recent researches are here fully described. At the present time there would appear to be some laxity in the use of Aeronautical terms. Throughout the book the technical terms and symbols used are those suggested by the Royal Aeronautical Society of Great Britain.

As far as possible each chapter of the present work has been made complete in itself. Chapter I presents fairly completely the Aerofoil Theory of the Airscrew, which is now commonly accepted as the principal theory of design. Other airscrew theories are described in outline in Chapter II. A general discussion of the nature of the air-flow around an airscrew and also a quantitative analysis of the energy account of an airscrew are given in Chapter III.

A résumé of the Principle of Dynamical Similitude as applied to the particular problem of the airscrew will be found in Chapter IV. In Chapter V some standard methods of measuring the performances of both model and full-scale airscrews are described. Chapter VI is exclusively devoted to a general consideration of the performance of a typical airscrew. In Chapter VII are collected the data obtained from experiments with several typical airscrews ; a general discussion of the mutual interference of an airscrew and an aeroplane has been based on these experimental data. Tandem airscrews are examined, from both the theoretical and experimental standpoints, in Chapter VIII. In the following chapter attention is directed to the stressing and distortion of airscrew blades under load. The vibrations of an airscrew are dealt with at some length in Chapter X. Under the heading of Chapter XI are included several airscrew problems of outstanding interest. Windmills and helicopters are the subjects of Chapter XII.

For convenience of reference, a list of the symbols used throughout the work is appended. A bibliography of the subject is also given at the end of the book.

Although many specific references and acknowledgments are made in different parts of the work, the author feels that his special thanks are due to the Advisory Committee for Aeronautics for the permission to publish most of the subject matter. Also to Sir Richard Glazebrook, F.R.S., late Director of the National Physical Laboratory, and Dr. Stanton, F.R.S., Superintendent of the Aerodynamics Division, the author desires to acknowledge his appreciation of the facilities granted to him at the Laboratory.

A. F.

TEDDINGTON,
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CHAPTER I

THE AEROFOIL THEORY OF AN AIRSCREW

THE AEROFOIL THEORY OF AN AIRSCREW CONSIDERED IN OUTLINE

ALTHOUGH modern airscrew theory is sufficiently valuable to enable an experienced designer to calculate with good accuracy the performance of an airscrew, the problem in its real essence is one of great complexity. It is in fact quite impossible to get an *exact* mental conception of the air-flow around an airscrew, and à priori to express completely in mathematical language the true working régime. Where a rigid mathematical solution has been attempted, the mathematician has of necessity modified the physical aspect of the problem, or rather his conception of the physical aspect, with the consequence that the assumptions made impair somewhat the final solution. Any airscrew theory to be of value must in the main be substantiated by experiment, and it would seem that it is in the domain of experiment that we must look for any subsequent development of this subject. At the same time it must be realised that any advancement from the present state of knowledge must be somewhat slow because of the difficulty of measuring and interpreting the experimental data of a problem which involves so many variables. The study of how an airscrew works is very interesting—perhaps fascinating—and has attracted the attention of many theorists and experimenters. At the outset we shall consider the aerodynamic theory of an airscrew in outline only. Comments of both a critical and an elucidatory nature, as well as the general history of the development of the subject, are given later.

In the present aerodynamic theory of an airscrew the blade is considered to be an aerofoil of a form suitable to glide in a helical path, the performance being calculated from the forces acting on elemental strips of the blade. The elemental forces are calculated from aerofoil data obtained in a wind channel, but as will be shown later, the wind channel experiments do not take into consideration all the conditions under which the element of the blade works. The air-flow around each blade element is considered to be two-dimensional and uninfluenced by the presence of the neighbouring parts of the blade. Also the air is assumed to be flowing into the airscrew with a uniform axial velocity, so that pulsations and any radial or circumferential velocities are neglected.

It is of course impracticable to consider any irregular rotational motions

or eddies, other than those which are common to aerofoil flow, of which account is automatically taken when measuring the forces on an aerofoil in the channel.

We shall now consider the motion of, and the air forces on, an element of the blade of small width—such an element as shown in Fig. 1, but before doing so it is necessary to define the nomenclature to be used in the investigation.

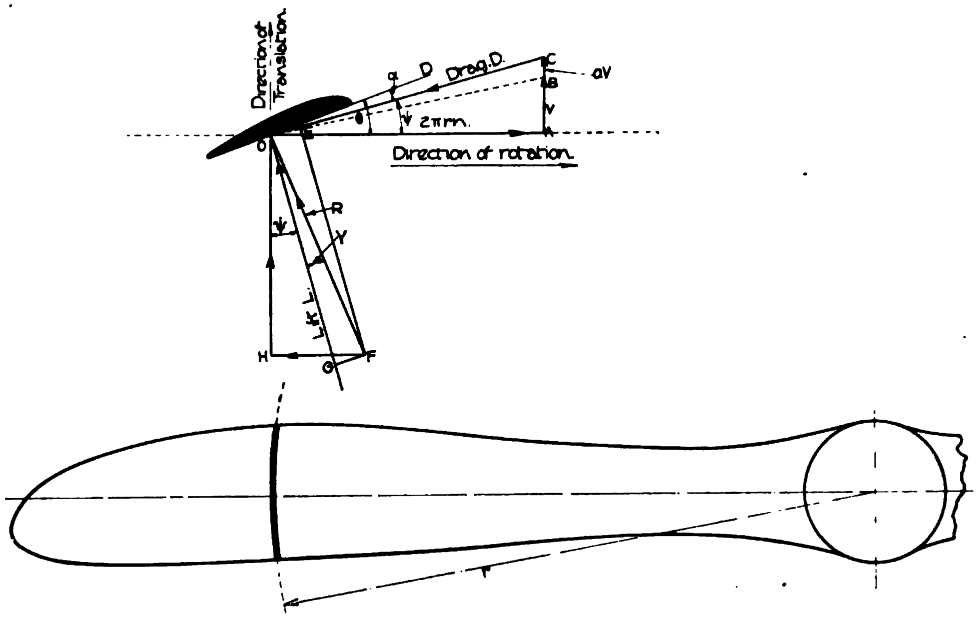


FIG. 1.

Let $V \equiv$ forward speed of the airscrew relative to the undisturbed air.

$aV \equiv$ the inflowing speed of the air at the front of any blade element relative to the undisturbed air.

Then $(1+a)V \equiv$ the speed relative to the airscrew of the air immediately in front of the blade element.

$n \equiv$ the rotational speed of the airscrew.

$r \equiv$ the distance of the blade element under consideration from the axis of rotation.

$\theta \equiv$ the blade angle of the section, that is, the angle between the chord of the section and a plane at right angles to the axis of rotation.

$\psi \equiv$ the angle between the direction of the relative wind at a blade section and a plane at right angles to the axis of rotation.

$\alpha \equiv$ angle of incidence of a blade section.

$L \equiv$ lift force acting on the blade section at an angle of incidence α.
$D \equiv$ drag force acting on the blade section at an angle of incidence α.
$\frac{L}{D} \equiv \text{Cot } \gamma =$ aerodynamic efficiency of a blade section.

It is fundamental that the difference between the two velocities, represented by V and aV, should be fully understood. As defined above " V " is the forward speed of the airscrew relative to undisturbed air, and so corresponds to the speed of an aeroplane on a calm day. On the other hand, " aV " is the speed of the inflowing air at the front of any blade element measured relative to the undisturbed air, and represents the motion imparted to the air due to the working of the airscrew. We may consider this inflowing velocity " aV " at any element of the blade to consist of two parts, one due to the disturbance of the blade element itself, which is of course a concomitant effect of the flow around an airscrew, and the other due to the interferences of the other blades and also of the blade itself. If the data of aerofoils tested separately in a wind channel are used in the calculation of the performance of the airscrew then it is the magnitude of the second part of the inflowing velocity, aV, which should be used, since the former is automatically taken account of in the wind channel experiments. Unless otherwise stated it is assumed, as far as the present theory is concerned, that the lift and drag forces taken for a blade element are those of a " free " aerofoil so that the symbol " aV " represents the inflow velocity at the blade element which is due to the interferences of the other blades and also of the blade itself. The subject of " inflow velocity " is of fundamental importance and will be considered in detail later.

Referring again to Fig. 1, it is seen that if the airscrew were rotating without any forward motion the blade element would be moving in the direction OA with a speed $2\pi rn$, so that if the air were stationary the angle of incidence of the element would be DOA, that is, the blade angle θ. If in addition to the rotational motion the speed relative to the airscrew of the air immediately in front of the blade element is $(1+a)V$—represented by the line AC—then the element is moving through the air in the direction OC with an angle of incidence DOC. The angle COA has been denoted by the symbol ϕ, so that $\tan \phi = \frac{(1+a)V}{2\pi rn}$. Also $\alpha = (\theta - \phi)$.

Utilising the analogy of the aerofoil, we may now proceed to develop the theory of an airscrew.

Turning our attention to one element of the blade—all the other elements may be similarly treated—we have already seen that owing to the combination of the translational and rotational motions relative to the air, the blade element may be regarded as an aerofoil moving in a helical path of angle ϕ, where $\tan \phi = \frac{(1+a)V}{2\pi rn}$, with a speed relative to the air of

$$[(V+aV)^2+(2\pi rn)^2]^{\frac{1}{2}},$$

the angle of incidence α being $(\theta - \phi)$.

If the aerodynamic properties of this aerofoil shape are known it is possible to calculate for the known values of the speed and of the angle of incidence, the lift force, L, and the drag force, D, acting on the blade element.

In the figure the lift L and the drag D are represented in magnitude and direction by GO and EO respectively, so that the resultant force on the section is represented in magnitude and direction by FO. The component of this resultant force in the direction of the forward motion ($L \cos \psi - D \sin \psi$)—represented in magnitude and direction by HO of Fig. 1—gives the contribution of the blade element towards the thrust of the airscrew. The component in the direction of the rotational motion ($D \cos \psi + L \sin \psi$)—represented in magnitude and

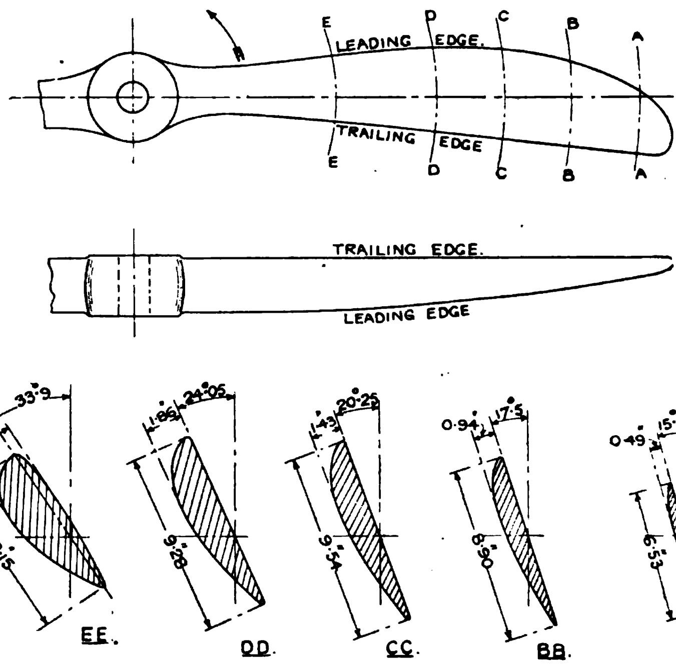


FIG. 2.

direction by FH—gives the force opposing the rotation of the element, and when multiplied by the distance of the element from the axis of rotation gives the torque needed to drive the element through the air. The thrust and torque of the airscrew may be obtained from the integration of the thrust and torque of each element of the blade. Most of the thrust is developed, and most of the torque is expended, at the outer portions of the blade which are moving at the greater velocity, the maximum values of the thrust and torque occurring at a distance from the airscrew axis of about three-quarters of the length of the blade.

The useful work done per second is the product of the thrust ($L \cos \psi - D \sin \psi$) and the forward velocity V. The work done per second in driving the blade element through the air is the product of the force ($D \cos \psi + L \sin \psi$), opposing the rotation, and the distance $2\pi rn$ moved against this force.

The efficiency of working is therefore equal to

$$\frac{V(L\cos\psi - D\sin\psi)}{2\pi rn(D\cos\psi + L\sin\psi)},$$

that is, $$\frac{1}{(1+a)}\,\frac{\tan\psi}{\tan(\psi+\gamma)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $\tan\gamma = \frac{D}{L}$.

It is seen from the above expression that the efficiency of working of a blade element depends on three factors: (*a*) The aerodynamic efficiency $\left(\frac{L}{D}\right)$, (*b*) the pitch angle ψ, and (*c*) an inflow factor, $\left(\frac{1}{1+a}\right)$.

Each of these three factors is considered in detail later.

Calculation of the Performance of an Airscrew

To understand thoroughly the preceding aerodynamic theory it is necessary to calculate in detail the performance of an airscrew. A sketch of the airscrew for which the calculations are to be made is shown in Fig. 2. This two-bladed airscrew of diameter 9 ft. was designed to develop at ground level a thrust of 625 lb. when moving through the air with a forward speed of 104 m.p.h. and a rotational speed of 1500 r.p.m. The form and size of the blade are defined completely by the data of the following table.

TABLE I

Data of Blade Form

Diameter of airscrew = 9 ft.

Distance of section from the axis of rotation. r feet.	Length of chord of section. C feet.	Ratio of the maximum thickness to chord length.	Maximum thickness. Inches.	Blade angle θ. Degrees.
4·22	0·545	0·075	0·490	15·50
3·66	0·740	0·105	0·935	17·50
3·10	0·795	0·150	1·430	20·25
2·53	0·770	0·200	1·850	24·05
1·69	0·680	0·300	2·450	33·90

We are now going to calculate the performance of this airscrew when moving through the air at ground level with a forward velocity of 104 m.p.h. and a rotational speed of 1500 r.p.m. Before this may be done, however, it is necessary to estimate the probable magnitude of the inflow velocity aV for each element

of the blade. Since the primary object of the present calculation is one of illustration, it is thought that an approximate method of calculating the magnitude of this inflow velocity is perhaps sufficient. For this purpose, then, the air is assumed to be flowing with a uniform velocity into the airscrew disc, the magnitude of aV being calculated from the "Froude" expression $T=\frac{\pi D^2}{4}\rho V^2(1+a)2a$,* where $T=625$ lb., $D=9$ ft., $\rho=0{\cdot}00237$ slugs per cubic foot, and $V=152{\cdot}5$ ft. per sec. Hence $aV=12{\cdot}6$ ft. per sec. The appropriateness of this expression is considered elsewhere. Also it will be shown later that the magnitude of aV varies along the blade. It has been the author's experience that this approximate method of estimating the magnitude of aV enables the calculations of the airscrew performance to be made with fairly good accuracy.

Relationship between the Translational and Rotational Speeds at a Constant Thrust of an Airscrew.

Diameter of the Airscrew = 9 ft.

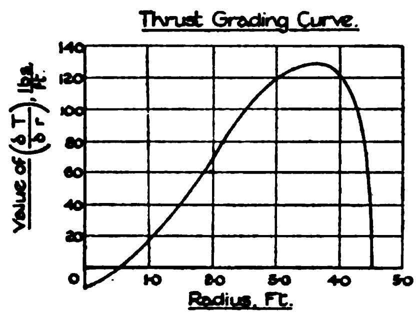


Fig. 3.

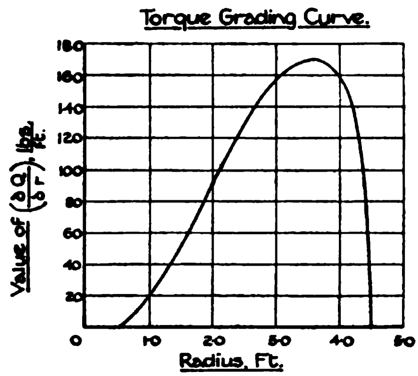


Fig. 4.

Proceeding in the manner indicated by the theory of the preceding pages, the performance can now be readily calculated. The value of ψ at each section of the blade is directly obtained from the expression $\tan\psi=\frac{(152{\cdot}5+12{\cdot}6)}{50.\pi.r}=\left(\frac{1{\cdot}05}{r}\right)$; also the angle of incidence α from the expression $\theta=(\psi+\alpha)$.

Utilising the aerodynamical data of the blade sections, which are obtained directly from wind-channel experiments, calculations of both the lift and drag forces acting on a blade element are then made.

Thus the lift, $L=k_L\times\rho\times\text{area}\times(\text{wind speed})^2=k_L.\rho.C.\delta r[(V+aV)^2+(2\pi rn)^2]$, and the drag, $D=k_D\times\rho\times\text{area}\times(\text{wind speed})^2=k_D.\rho.C.\delta r[(V+aV)^2+(2\pi rn)^2]$, where k_L and k_D are the absolute coefficients of lift and drag respectively for the blade section at an angle of incidence α.

From the expressions on p. 4 it is seen that the thrust δT, contributed by an element of the blade of chord length C, and of width δr is $C.\delta r.\rho[(V+aV)^2+(2\pi rn)^2]\times[k_L\cos\psi-k_D\sin\psi]$,

that is $k_L.\rho.C.\delta r.\frac{(V+aV)^2}{\tan\psi\sin\psi}\left[1-\frac{k_D}{k_L}\tan\psi\right]$.

* See p. 22

Also the torque δQ, needed to drive the element through the air, is
$C.r.\delta r.\rho[(V+aV)^2+(2\pi rn)^2][k_D \cos\psi+k_L \sin\psi]$,

that is, $k_L.\rho.C.r.\delta r \dfrac{(V+aV)^2}{\tan\psi \sin\psi}\left[\tan\psi+\dfrac{k_D}{k_L}\right]$.

The thrust and the torque of the airscrew are then calculated from the integration of the thrust contributed by, and the torque expended on, each element of the blade. The data of these calculations are given in detail in Table II, which illustrates the method of procedure. The curves of Figs. 3 and 4 have been plotted from the data of this table and show how the thrust and the torque of each element of the blade vary with the distance of the element from the axis of rotation.

An inspection of these curves shows that the outer part of the blade which is moving through the air at the greater speed is doing most work. The maximum thrust is developed and the maximum torque absorbed by the part of the blade which is at a radial distance of about 0.8R from the axis of rotation. From the thrust-grading curve it will be seen that the central part of the airscrew makes a negative contribution towards the thrust of the airscrew.

The thrust developed by each airscrew blade is obtained directly from the area of the thrust-grading curve.

It is seen from Table II that the thrust and the torque of this two-bladed airscrew are 626 lb. and 816 lb.-ft. respectively.

Also the efficiency of the airscrew $=\left(\dfrac{VT}{2\pi Qn}\right)100$ per cent

$$=\frac{104\times 88\times 626\times 100\times 7}{44\times 814\times 1500}=74{\cdot}8 \text{ per cent.}$$

Finally, it is of interest to notice that the value of the preceding calculation depends firstly on the accuracy of the method of estimating the magnitude of the inflow velocity, and, secondly, on the extent to which the values of k_L and k_D of the calculations differ from the values of practice.

Some Notes on Airscrew Design

Although not wishing to venture too far beyond the scope of the present work, the author feels that a few remarks on the method of designing an airscrew may not be out of place. The design of an airscrew is not exclusively an aerodynamical problem. The exigencies of practice also need consideration. For example, if the diameter be large the landing gear may become unduly heavy and cumbersome ; whilst on the other hand, the diameter must be sufficiently large to prevent the ratio of the disc area of the airscrew to the area of the obstructions, either in front or behind, as the case may be, from being small. Also the stresses in the mateiial of the airscrew should not be too great. The problem presented to the designer is to design an airscrew which when driven by a particular engine will give an aeroplane, of known type, the best possible performance, either of climb or of maximum speed, as the case may be. It follows then that the airscrew cannot be considered as an independent entity, since its performance

TABLE II

Calculation of the Performance of the Airscrew

Forward speed of the machine = 104 m.p.h.

Rotational speed of airscrew = 1500 r.p.m.

Distance of section from axis of rotation. r feet.	Length of chord of section. C feet.	θ Degrees.	ψ Degrees.	α Degrees.	k_L	$\left(\frac{k_L}{k_D}\right)$	Value of $\frac{k_L.\rho\, C.(V+aV)^2}{\sin\psi\tan\psi}$	Value of $\left(1-\frac{k_D}{k_L}\tan\psi\right)$	Value of $\left(\frac{k_L}{k_D}+\tan\psi\right)$	Value of $\left(\frac{\delta T}{\delta r}\right)$ (lb./ft.).	Value of $\left(\frac{\delta Q}{\delta r}\right)$ lb.
4·22	0·545	15·50	14·00	1·5	0·185	16·4	107·5	0·985	0·310	106·0	140·2
3·66	0·740	17·50	16·00	1·5	0·220	16·2	133·1	0·982	0·349	130·5	170·0
3·10	0·795	20·25	18·75	1·5	0·265	13·8	124·5	0·975	0·412	121·0	160·0
2·53	0·770	24·05	22·55	1·5	0·340	13·5	106·5	0·969	0·489	103·0	132·0
1·69	0·680	33·90	31·90	2·0	0·370	11·5	49·5	0·946	0·709	46·7	59·2

Thrust of airscrew $=2\times313=626$ lb.

Torque ,, $=2\times408=816$ lb.-ft.

depends on the characteristics of both the engine and the aeroplane. In fact, the principal parameters of the design, such as forward speed, rotational speed, and either the thrust or the torque, are fixed by the working conditions imposed by the characteristics of the engine and of the aeroplane. The remaining parameters, such as diameter, plan form, number of blades, shape of blade sections, depend to some extent on the discretion and the experience of the designer. Assuming the values of V, n, and D are known, the problem is to adjust both the plan form of the blade and the angle of incidence of each blade section so that the given thrust may be developed, or the given torque expended, as the case may be, most efficiently. Usually, the linear scale of the chord-length of a blade section is the unknown quantity of the design, the angle of incidence of each section being that at which the maximum aerodynamic efficiency for the section is developed.

Parenthetically, it is worth noting that if a mean inflow velocity be taken in the design, then in practice the angle of incidence of a blade section in the neighbourhood of the working part of the blade will be slightly less and in the neighbourhood of either the boss or the tip slightly greater than that of the design, because the inflow velocity in the neighbourhood of the blade where most of the thrust is developed is greater than the average value. Appropriate adjustments of the blade angles are, however, easily made by an experienced designer.

Having decided on some suitable plan form, each blade section is drawn in at the correct blade angle. The contour lines of the plan form are then constructed. After fairing the contour lines it may be necessary to refair the sections. By such a tentative method of fairing—that is, constructing the contour lines of the plan form from the sectional shapes, and vice versa—we obtain eventually a smooth blade surface. The contour lines should not suddenly undergo any great changes of curvature, otherwise the blades may, by virtue of their great twist, present an unsightly appearance. The success of the above operation, like all fairing processes, depends greatly upon the experience and the practical instinct of the designer. A study of the characteristics of aerofoil sections of different shape should enable the designer to approximate with some accuracy to the aerodynamical properties of those sections, the shapes of which have been slightly modified by the fairing. It has been the author's experience that the fairing of the blade surfaces, if well performed, does not greatly alter the shape of the blade sections.

Finally, during the design two points should be borne in mind : (*a*) the outer blade sections should be of good aerofoil shape because they are doing most work, and (b) the shapes of the sections at the boss of the airscrew are determined principally from considerations of strength.

The Aerofoil Theory, as Originally Developed

The conception that the performance of an airscrew may be calculated from the integration of the forces acting on elemental strips of the blade, each blade element being regarded as an aerofoil moving in a helical path, is due to

Drzewiecki.* Lanchester† independently published the same theory in 1907. In these early theories only the motion of the blade relative to the undisturbed air is considered; the inflow velocity due to the working of the airscrew being neglected.‡ In 1909 Drzewiecki published "Des Hélices Aériennes," in which he develops the blade-element theory based on a method adopted by him in a note presented in 1892 to the Association Technique Maritime. A method of calculating the performance of an airscrew having a constant angle of incidence of the optimum value, which is taken as 2°, is given in some detail in this book. The plan form of the blade was of a rectangular shape of width 1/6th of the length, so that the performance could be readily calculated algebraically. The limitation of the length to six times the width is of course unnecessary, and in a later note Drzewiecki develops more general formulæ in which this ratio is a parameter.

Some experiments by Dorand§ in 1910 show that the theory, which assumes that the maximum efficiency of an airscrew is developed when the blade sections are working at a constant optimum angle of incidence, does not hold completely in practice. Dorand comes to the conclusion that account should be taken of the inflowing motion of the air at the front of the airscrew, since with the assumption of no inflow velocity the angle of incidence, as calculated, overestimates the value of practice. An experimental investigation of the accuracy of the initial assumptions of the early aerofoil theory of an airscrew was made at the National Physical Laboratory by Bramwell|| and Fage. As already stated, two of the assumptions of this theory are that the force on a blade element is due directly to the velocity relative to *still* or undisturbed air and also that the blade elements may be treated independently of each other.

These experiments showed that the angle of incidence which gave the highest value of the ratio of lift to drag for an aerofoil tested independently in a wind channel was not necessarily the best angle of incidence for the aerofoil shape when used in the design of an airscrew. Further, we found a considerable discrepancy between the performance of an airscrew as measured experimentally, and the performance as calculated from the integration of the elemental forces on the blade, when such forces were estimated from aerofoil data measured directly in a wind channel and the inflow velocity was neglected. The value of the latter series of experiments suffers somewhat because the aerofoil data were obtained at a wind speed of only 30 ft. per sec., whereas the resultant velocities of some of the blade sections were very high. The final conclusions we made were (*a*) that the character of the air-flow around the blades was probably different from that assumed—e.g. no account was taken of the inflow velocity—and it was perhaps a matter of surprise that the agreements between the calculated

* "Des Hélices Aériennes. Théorie générale des propulseurs hélicoidance et methode de calcul de ces propulseurs, pour l'air," S. Drzewiecki. Published 1909.

† "Aerodynamics," F. W. Lanchester. Constable & Co. Ltd., 1907.

‡ Lanchester in a later theory takes account of the inflow velocity.

§ "Étude expérimentale des hélices propulseurs," E. Dorand. "La Technique Aéronautique," Nov., 1910.

|| "Experiments on the effect of varying the angle of incidence of the blades of a propeller, with a comparison between the actual performance of a propeller and that calculated with the aid of data obtained from experiments on aerofoils of appropriate sections," F. H. Bramwell, B.SC., and A. Fage, A R.C.SC. Adv. Comm. Aeron., 1913.

and measured performances were so good, and (*b*) that although the method was not correct it was perhaps the best that could then be used in the design of an airscrew.

Dependence of the Efficiency of a Blade Section on the Values of (L/D) and ψ

It has already been shown that the efficiency η of a blade element may be expressed as $\dfrac{1}{(1+a)}\dfrac{\tan\psi}{\tan(\psi+\gamma)}$. The efficiency is therefore a function of (*a*) the aerodynamic efficiency (L/D), (*b*) the angle ψ, which is calculated from the motion of the element relative to the air *immediately* in front, and (*c*) the inflow factor $\dfrac{1}{(1+a)}$.

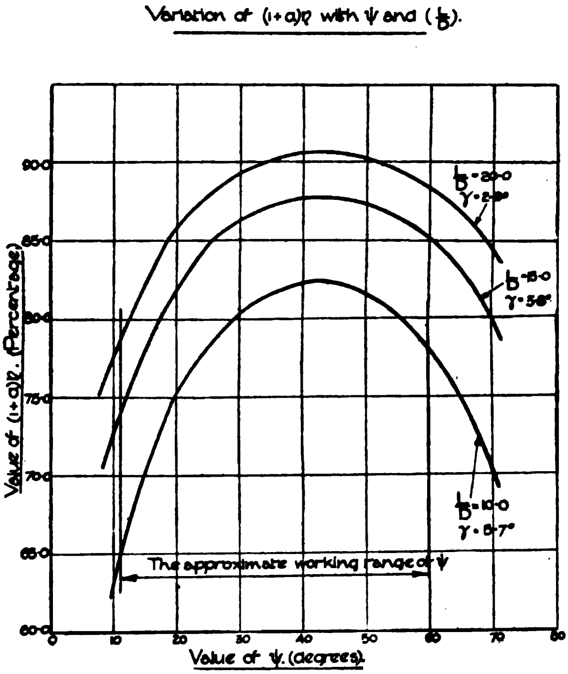


Fig. 5.

To consider the influence on η of the first two parameters, namely (L/D) and ψ, calculations of efficiency have been made for three different types of aerofoil section, working at such attitudes relative to the wind that the values of (L/D) are 20·0, 15·0, and 10·0 respectively. The corresponding values of γ are 2·9°, 3·8°, and 5·7° respectively. With each shape of aerofoil the value of ψ was varied from 10° to 70°. The data of these calculations are shown graphically in Fig. 5. When considering the variation of η with ψ at a constant value of γ, the value of "a" is regarded as a constant, from which it follows since ψ is a function of "a" that the comparisons are made at different values of both (V/n) and thrust. This is, however, quite immaterial to the present discussion.* It can be easily shown mathematically that the expression $\dfrac{1}{(1+a)}\dfrac{\tan\psi}{\tan(\psi+\gamma)}$ has its maximum value when $\psi=(45-\gamma/2)^0$, the value of "a" being maintained constant by suitable adjustments of V, n, and T. If, therefore, we could design an airscrew to develop most of its thrust at the radius which for the particular forward and rotational speeds makes ψ about 45° we could, by using blade sections which were very efficient aerodynamically, obtain a very high efficiency for the airscrew. Unfortunately, this is quite

* The inappropriateness of such an assumption to the case of a blade element of a particular airscrew is considered elsewhere.

impracticable, since such an airscrew with the rotational and forward speeds of practice would, owing to its small diameter, develop a very small thrust. The importance of employing in the design of an airscrew, blade sections of high aerodynamic efficiency is shown very clearly by the curves of Fig. 5. Considering 11° to 45° as the important part of the working range of ψ, the value of $(1+a)\eta$ only falls from 90·5 per cent to 79·0 per cent when $L/D=20{\cdot}0$. With $L/D=10{\cdot}0$ the value of $(1+a)\eta$ falls not only at a greater rate (82·5 per cent to 64·5 per cent), but its value over the greater part of the " ψ " range is lower than the minimum of 79·0 per cent when $(L/D)=20{\cdot}0$.

Hence to design an airscrew of good efficiency the blade sections should have high values of (L/D). The value of ψ has a smaller influence on the efficiency than the aerodynamic efficiency (L/D) of the blade section. Also, whenever possible it is advantageous to have large blade angles at the outer part of the blade, which contributes most towards the thrust of the airscrew because of the greater speed.

We have seen then that the maximum efficiency of a blade element—regarded as a separate entity—is obtained when the angle of incidence is such that the maximum aerodynamic efficiency, (L/D), is developed for the particular aerofoil shape. The problem of design, however, is to obtain the maximum possible efficiency of the airscrew.

At first sight it would appear that the obvious solution is to choose for each blade section the angle of incidence which gives the maximum value of L/D. In this case the value of $(1+a)\eta$ of the blade section is a maximum. It has been pointed out by Berry* that this method is not completely sound and would, in fact, only be true if it were possible to alter the efficiency of a section without changing the thrust, or if the efficiency of each section were the same. Neither of these conditions is realised in practice. Berry shows, theoretically, that the overall efficiency of an airscrew could be increased if, instead of working at the angle of incidence which for each section makes the value of (L/D) a maximum, the design is such that the angle of incidence of a section at the boss and tip is smaller, and for an intermediate section larger than the best value. In view of the somewhat lengthy calculations involved it would be a tedious matter to so design an airscrew. Moreover, the resulting gain of maximum efficiency would probably be quite small. Some slight arbitrary adjustment of the angle of incidence in the manner suggested by Berry could be made, however, without any undue labour.

Betz† found that an airscrew has the best efficiency when the angle of incidence of a blade section is about 3° or 4°, the angle increasing slightly the nearer the blade section to the tip. He also found that an airscrew, with which the under-surface formed a true screw surface was not the most efficient.

* "Note on the efficiency of an airscrew," A. Berry, M.A. Advisory Committee for Aeronautics.

† "Systematische Versuche an Luftschrauben—Modellen A. Betz." Jahrbuch der Luft-Fahrzeug-Gesellschaft, 1912–13.

Influence of the Inflowing Motion of Air on the Performance of an Airscrew

A fundamental shortcoming of the early aerofoil theory of an airscrew, was the omission to take account of the inflowing motion of air at the front of the airscrew. We have already seen that if the inflow velocity be neglected when designing an airscrew the calculated values of both the efficiency and the thrust are greater, at the same forward and rotational speeds, than those realised in practice. That is, the angles of the blade as designed are too small. The importance of considering the inflow velocity during a design may be easily demonstrated by some simple calculations of the performance of a blade element of a helicopter, that is, an airscrew rotating at a stationary point. The case of a helicopter, which is admittedly an extreme one, has been taken for illustrative purposes only. We shall assume the helicopter to be rotating at about 1500 r.p.m., and also the blade element under consideration to have an area of 6 sq. in. at a radial distance of 3 ft. from the axis of rotation. The blade angle of the element will be taken as 5°.

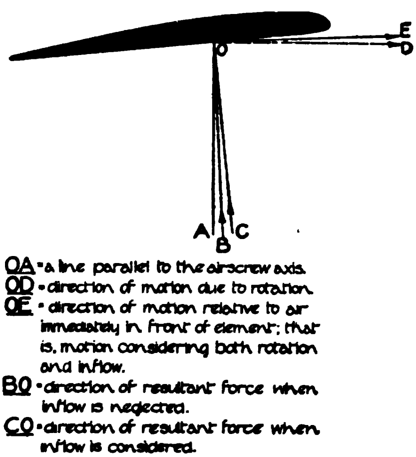


Fig. 6.

With such working conditions the thrust and torque of this blade element as calculated from the aerofoil theory which neglects the inflowing motion of the air were 8 lb. and 1·2 lb.-ft. respectively. It will be seen from Fig. 6 that in this case the resultant force acting on the element is in the direction BO, where OD is the direction of the motion of the element which in this hypothetical case is due to the rotation only. In practice there is, however, an inflow velocity of air, so that the motion of the element relative to the air immediately in front is in the direction OE. The angle DOE for the assumed working conditions is about 2·5°. The resultant force on the section has now the direction CO. With these practical conditions of working, the thrust and torque as calculated from the aerodynamic data of the element are 5·5 lb. and 1·62 lb.-ft. respectively. It is seen then that when account is not taken of the inflowing velocity, the thrust in this particular case is overestimated by about 45 per cent and the torque is underestimated by about 26 per cent.

When the airscrew has a forward motion in addition to the rotational motion, the influence of the inflow velocity on the performance, although not so pronounced as in the case of the helicopter, is still very appreciable. Taking a mean value over the working ranges of a large number of airscrews, the writer has found that the ratio of the thrust, as calculated from the aerofoil theory which does not take into consideration the velocity of the inflowing air, to the actual experimental thrust is of the order of 1·35, the corresponding

ratio for the torque being about 1·25. The values of both these ratios are, of course, dependent on the value of the forward advance per revolution at which the airscrew works.

An Aerofoil Theory in which the Inflow Velocity is Calculated Empirically

It is somewhat unfortunate that at the present time there is no exact method of calculating or of measuring the magnitude of the inflow velocity, aV, which, as we have seen, is one of the most important parameters influencing the performance of an airscrew. A method of calculating empirically the magnitude of this inflow velocity has been developed at the National Physical Laboratory by Fage* and Collins. This theory was first presented to the Advisory Committee for Aeronautics in September, 1916. A similar theory which is considered later has been developed independently by Riach.†

The basis of the Fage-Collins Theory is to adjust the magnitude of the inflow velocity at any blade element so that the thrust of the element as calculated from considerations of momentum is in agreement with the thrust, as calculated from the aerofoil theory, whilst at the same time the thrust of the airscrew as calculated from the integration of the elemental thrusts is in good agreement with the thrust of practice. From p. 6 it is seen that the thrust of a blade element as calculated from aerofoil data is

$$\rho.C.dr(2\pi rn)^2.k_L.\sec\psi\left[1-\frac{k_D}{k_L}\tan\psi\right],$$

where $\tan\psi=\dfrac{(1+a)V}{2\pi rn}$.

As already stated, aV is the inflow velocity immediately in front of the blade element, and is exclusive of any inflow velocity which is common to the ordinary aerofoil flow. Also the magnitude of aV differs from that of the mean inflow velocity into the annulus described by the blade element. *Assuming*, however, that the inflowing velocity into the annulus is uniform and of magnitude aV—the greater the number of blades the more nearly uniform will be the inflow velocity of the air—the mass of air flowing through the annulus would be $2\pi r dr\rho.(1+a)V$, so that the thrust of the element may be expressed by $2\pi.r.dr\rho(1+a)abV^2$, where ab is an empirical constant. From considerations of momentum abV may, in some respects, be regarded as the total augmented velocity of the air in the outflowing stream due to the working of the blade element. It should here be stated, however, that the conception underlying the present method of calculating the magnitude of the inflow velocity, aV, is admittedly of a hypothetical nature; the analogy with the momentum theory

* "An investigation of the magnitude of the inflow velocity of the air in the immediate vicinity of an airscrew, with a view to an improvement in the accuracy of prediction from aerofoil data of the performance of an airscrew," by A. Fage, A.R.C.SC., and H. E. Collins. Advis. Comm. Aeron., 1917.

† "The Screw Propeller in Air," M. A. S. Riach, A.F.AE.S. "Journal of the Royal Aeronautical Society," 1917.

of Froude* should not be carried too far. Equating the "aerofoil" and "momentum" thrusts we obtain the expression

$$(1+a)a = \frac{N_B.C.(2\pi rn)^2.k_L \sec\psi}{2\pi.r.b.V^2}\left[1 - \frac{k_D}{k_L}\tan\psi\right]$$

where N_B = the number of blades.

If, then, the aerodynamic data of the blade section, the working speeds of the airscrew, and also the value of "b" are known, the value of "a" may be directly calculated from the above expression. Accordingly, then, an attempt was made to determine a value of "b" which would be applicable to the calculation of the performance of any airscrew, when the aerodynamic data of the blade sections and also the working speeds were known.

The method of calculation was such that at any chosen value of (V/n) "b" was given an arbitrarily fixed value for all the blade sections, and the value of "a" for each blade section was found from the above expression. Then from these values of "a" and the assumed value of "b," the thrust of the airscrew was calculated directly from the aerofoil theory. At first this calculated thrust did not agree with the thrust as measured experimentally, but by successive approximation a value of "b" was eventually found which gave complete agreement. Using this final value of "b," the complete performance of the airscrew was calculated from the values of "a" calculated from the above expression and the aerodynamic data of the blade sections.

The value of this theory depends fundamentally on whether a representative or mean value of "b" can be determined which will allow the performance over the working range of (V/n) of any airscrew to be calculated with good accuracy. With a view to ascertaining whether this was possible, calculations were made for six airscrews of widely different characteristics. Both the performance and the aerodynamic data of the blade sections of each airscrew were known. The latter data were measured, at a high wind speed, on square-ended aerofoils of aspect ratio 6. We found that the value of "b" was a function of the characteristics of an airscrew: the mean value of "b" for the six airscrews being 2·1. It was thought of interest to consider what loss of accuracy in the calculations of the performance of any airscrew would result from using this mean value of "b," and accordingly further calculations with the data of the six airscrews were made. We then found that the thrust of an airscrew as calculated from this theory, in which the value of "b" was taken as 2·1, agreed over the working range of (V/n) within about 2 per cent with the thrust as measured experimentally. Further, the calculated torque agreed within $\pm$ 5 per cent with the experimental value.

The close agreement of the calculated and experimental values of the torque is probably fortuitous. There is no apparent reason why exact agreement should be obtained because since the calculated thrust is made to agree with the measured thrust, it follows that the discrepancy between the measured and calculated values of the torque would be a numerical measure of the deficiencies of the

* See later.

theory. The close agreement between these two values of torque—calculated and measured—may mean either that the assumptions of the theory are not appreciably in error, or else the quantitative effects of these assumptions, if appreciable, tend to neutralise each other.

It should be fully appreciated that " b " is not a physical quantity, since its value has been obtained empirically. At the same time the utilisation in the calculation of the performance of an airscrew of some such factor as " b " is quite legitimate, as long as the assumptions there made are the same as those made in the researches from which the value of " b " was determined. Thus, the effects of aspect ratio of the blade, blade shape, the interference of the neighbouring parts of the blade on the force on a blade element, etc., are taken into account empirically by the factor " b."

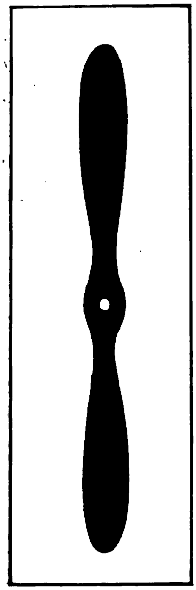


FIG. 7.

To illustrate the theory the performance of an airscrew, of which a photograph is given in Fig. 7, has been calculated, the assumed value of " b " being 2·1. The data of these calculations and also the performance data of the airscrew are given in Table III and shown graphically in Figs. 8 and 9. From a reference to the table it will be seen that over the working range of (V/nD), the calculated thrust differs from the measured thrust by only 1 per cent; also the discrepancy between the measured and calculated values of the torque is of the order of 4 or 5 per cent. With this particular airscrew it will also be noticed that calculations of the performance from the aerofoil theory in which the inflow velocity of the air is neglected, overestimate both the thrust and the torque by about 22 per cent and 7 per cent respectively.

The curves of Fig. 8 show for several values of (V/nD) how the value of "a" varies with the radial distance of the element from the axis of rotation. It is seen that "a" depends on both the position of the element in the blade and the value of (V/n) at which the airscrew works. Moreover, if (V/n) be constant, the value of " a " varies from zero at the tip to a maximum at a radial distance of about 0·7R from the axis of rotation, and then diminishes rapidly towards the boss. From the curves of Fig. 9 it would appear that the relationship between the value of "a" for any blade element and the thrust of the airscrew may be expressed with good accuracy by a linear equation. It is not to be expected that the velocity distribution at the front of the airscrew, as calculated from this theory, should represent exactly that of practice, since one is of an empirical and the other of a physical nature.

Although the present theory allows the performance of an airscrew to be calculated with good accuracy it cannot be applied directly in design because the form parameters of an airscrew—C, σ, and θ—are of course not known until the design has been completed. The author has found, however, that a very close approximation to the final form of an airscrew may be made if the air be assumed

TABLE III

PERFORMANCE DATA OF THE TWO-BLADED AIRSCREW OF WHICH A PHOTOGRAPH IS SHOWN IN FIG. 7

Diameter $=8$ ft.

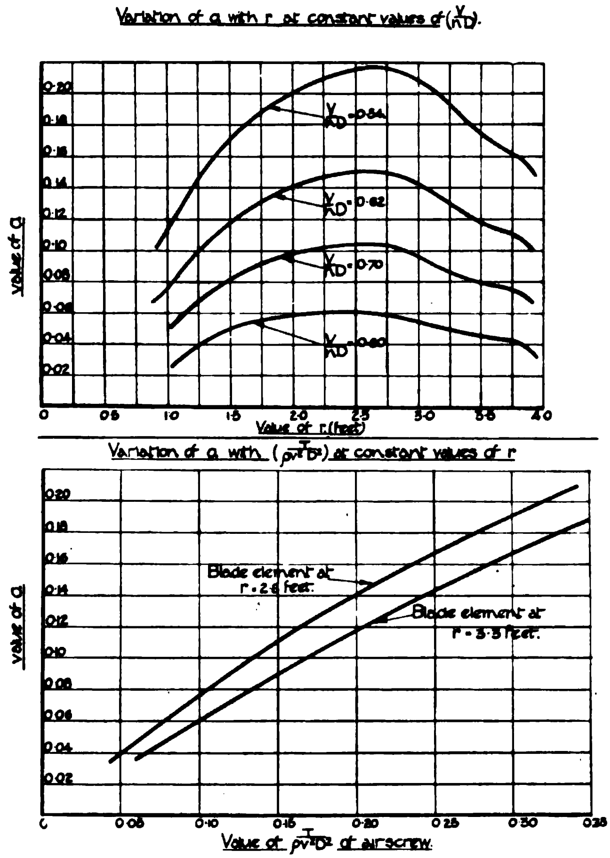
Value of $b=2{\cdot}1$.

Value of (V/nD).	Value of ($T/\rho n^2D^4$)					Value of ($Q/\rho n^2D^5$)					Efficiency (Percentage)				
	As measured. (A)	Calculated from aerofoil theory in which inflow of air is neglected. (B)	Calculated from aerofoil theory *with* inflow of air. (C)	Ratio B/A	Ratio C/A	As measured. (D)	Calculated from aerofoil theory in which inflow of air is neglected. (E)	Calculated from aerofoil theory *with* inflow of air. (F)	Ratio E/D	Ratio F/D	As measured. (G)	Calculated from aerofoil theory in which inflow of air is neglected. (H)	Calculated from aerofoil theory *with* inflow of air (K)	Ratio H/G	Ratio K/G
0·80	0·0500	0·0580	0·0495	1·16	0·99	0·00885	0·00950	0·00835	1·07	0·95	72 0	77·2	75·1	1·07	1·04
0·70	0·0695	0·0850	0·0695	1·22	1·00	0·01090	0·01185	0·01040	1·09	0·96	71·3	80·0	74·5	1·12	1·04
0·62	0·0835	0·1045	0·0850	1·25	1·02	0·01200	0·01280	0·01150	1·07	0·96	68·5	81·3	72·8	1·19	1·06
0·54	0·0985	0·1210	0·0990	1·23	1·00	0·01290	0·01360	0·01225	1·05	0·95	65·5	76·3	68·3	1·17	1·04

to flow uniformly into the airscrew and the value of " a " is calculated directly from the expression

$$T = 1{\cdot}65\rho D^2V^2(1+a)a.$$

The merit of using this value of " a " in the preliminary design is that the blade angles are increased by about the right magnitude to take into account the inflowing motion of the air. Afterwards, if it be considered necessary, minor adjustments of blade angle of appropriate magnitude to allow for the varying inflow velocity may be made.



FIGS. 8 and 9.

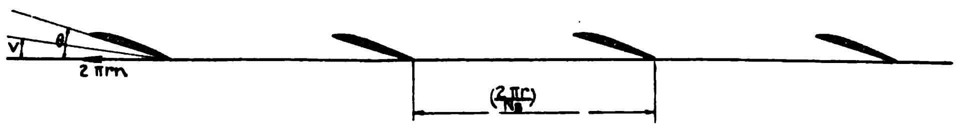
In conclusion, we found when developing the present theory, that both the thrust and torque as calculated from an aerofoil theory which neglects the inflow velocity are considerably greater at the same value of (V/nD) than the values as measured by direct experiment. Taking average values over the working range of (V/n) of the six airscrews, the ratio of the thrust, as calculated when aV = 0, to the thrust, as measured, was 1·29. Making a similar comparison with the torque, the ratio of the theoretical and experimental values was 1·15.

As stated elsewhere, a somewhat similar method of calculating the inflow velocity has been developed independently by Riach.* In this theory the thrust, as calculated from a momentum theory, is equated to the thrust as calculated from an aerofoil theory in which inflow velocity is considered, the value of " b " being taken as 2·0. It should, however, be pointed out that with this arbitrarily fixed value of " b " it does not necessarily follow that the calculated thrust will be in agreement with the measured thrust. With the Fage-Collins Theory the value of " b " for any airscrew was so adjusted that the thrust as calculated from either the aerofoil theory or a quasi-momentum theory was in agreement with the measured thrust. Riach has, however, shown that a further controlling parameter would be a rotational inflow velocity.

* Loc. cit.

A METHOD OF CALCULATING THE INFLOW VELOCITY AT A BLADE ELEMENT DUE TO THE INTERFERENCE OF THE BLADES

With the Fage-Collins Theory the magnitude of the inflow velocity is chosen so as to obtain agreement between the theoretical and practical performances of an airscrew. Also the inflow factor is of an empirical nature and embodies in itself the necessary correction to account for small errors due to the assumptions of the theory. It is of course obvious that to obtain agreement between the theoretical and practical values of both the thrust and the torque, two such empirical inflow factors, one of translation and the other of rotation, would be needed. It has been pointed out by Wood* and Glauert that the inflow velocity of air into an airscrew may be considered to consist of two parts—(*a*) the disturbance of air in front of the blade element under consideration, which corresponds to the ordinary disturbance of an aerofoil, and (*b*) the angular disturbances



N_B = number of blades.

FIG. 10.

of the other blades and of the blade itself at angular distances of $\pm 2\pi k$, where k s an integer. It is the latter part of the inflow velocity which should be used when calculating the performance of an airscrew, since the former is automatically taken into account when measuring the forces on an aerofoil in the wind tunnel. Further, the interference effect on one element of a blade due to the corresponding elements of the other blades is exactly analogous to the downwash and other interference effects experienced by one aerofoil of a series of equidistant aerofoils. Corresponding blade elements—elements at the same radii—will be uniformly spaced on a definite helix. Unrolling such a helix a multi-aerofoil series with a large negative stagger is obtained, as shown in Fig. 10.

Wood and Glauert considered the interference of the various blade elements to be represented by changes in the direction and the magnitude of the resultant velocity which determines the forces on the blade element under consideration. The method of obtaining experimental data was to test an aerofoil alone, and then as one of the series, the spacing of the aerofoils depending on the radial distance of the blade element from the axis of rotation and on the number of blades. From a comparison of the lift and drag curves obtained from the two experiments calculations were then made of the changes of the angle of incidence and of the relative air speed due to the interference of the other corresponding aerofoil elements of the series. A series of five aerofoils was used in these experiments; the measurements of lift and drag being made with the fourth aerofoil. The

* "Preliminary investigation of multiplane interference applied to airscrew theory," by R. McKinnon Wood and H. Glauert. Advis. Comm. Aeron., 1918.

magnitude of the inflow velocity, which should be used when calculating the airscrew performance from the aerodynamic data of aerofoils tested in free air, was then calculated from the experimental data. The data obtained from the experiments with the aerofoil under the interference of the remaining aerofoils of the series could of course be used directly in the calculation of the airscrew performance. A preliminary investigation on the lines outlined above shows (*a*) that the inflow correction to the rotational speed is of small importance, (*b*) that the ratio of the outflow and the inflow velocities, both taken relative to the undisturbed air, has not a constant value for all the blade elements.

A neglect of the rotational inflow velocity will of course necessitate a slight increase of the translational inflow velocity, if the force on the blade is to remain unchanged. If, however, the rotational inflow velocity be ignored, there is then only one parameter with which to obtain agreements between the theoretical and practical values of both the thrust and the torque. The outstanding merit of the present theory is that an attempt is made to determine in a logical manner the inflow velocities at the blade itself. It must be admitted, however, that such measurements of the performance of an aerofoil in a multi-aerofoil series do not consider all the conditions under which the blade element is working in practice; it would not be surprising then if some empirical correction factor were needed to make the theory agree with practice.

CHAPTER II

AIRSCREW THEORY CONSIDERED FROM OTHER STANDPOINTS

THE MOMENTUM THEORY OF R. E. FROUDE

FROUDE* regarded the screw as an advancing disc of instantaneous change of pressure, working in a fluid which was both inviscid and incompressible, so that the resistance of the established motion could be neglected. The dynamic operation of propulsion was considered to depend solely on the force due to the acceleration of the fluid, the whole of this acceleration taking place without the screw. The rotational motion in the outflowing column of the fluid was ignored. Fig. 11 gives a diagrammatic representation of the fluid flow around a screw rotating at a stationary point in a fluid which has a general velocity V.

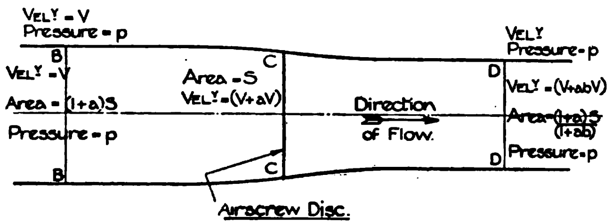


FIG. 11.

The motion of the inflowing column of fluid may be regarded as brought into being by a disc of negative pressure at the front of the screw, the flow being such that the total head at all points is a constant and equal to $\left(\frac{\rho V^2}{2}+p\right)$, where p is the static pressure and ρ the density of the undisturbed fluid. The fluid whilst passing through the screw has its total head increased, the increase which may be represented by H_0, being due to an increase of pressure head since the thick ness of the disc is very small, so that the velocity has not had time to change. The outflowing column of fluid contracts in area until its pressure is equal to p, the total head at any point in this column being equal to $\left(p+\frac{\rho V^2}{2}+H_0\right)$. The motion of the fluid surrounding both the inflowing and outflowing columns of air is such that the total head remains constant and equal to $\left(\frac{\rho V^2}{2}+p\right)$. The boundary of the outflowing stream at which the difference of total head is equal to H_0 is, therefore, a surface of discontinuity. Furthermore, the boundary surface between both the inflowing and outflowing columns of fluid is considered

* "On the part played in propulsion by differences of fluid pressure," R. E. Froude. "Trans. I.N.A.," 1889.

to be so shaped that the average pressure on the boundary is equal to the mean pressure, p, of the undisturbed fluid.

It is now a simple matter to calculate the thrust of the screw.

Let S = disc area of the screw.

ρ = density of the fluid.

T = thrust of the screw.

V = velocity of undisturbed fluid—the airscrew is regarded as rotating at a stationary point in a moving fluid.

$(1+a)V$ = velocity of the inflowing column of fluid taken relative to and at the disc of the screw.

$(1+ab)V$ = velocity relative to the screw of the outflowing column of fluid at the section where the pressure is p.*

Then, the mass of fluid passing through the disc of the screw $=\rho S(1+a)V$.

Also the total energy imparted to the fluid column $=\frac{\rho S(1+a)V^3}{2}\left[(1+ab)^2-1\right]$,

and since no work is done at the boundaries of the inflowing and outflowing column, this expression must be equal to $TV(1+a)$, that is, the work done at the disc of the screw, so that

$$T=\frac{\rho SV^2}{2}\left[(1+ab)^2-1\right]=\frac{SV^2ab}{2}(2+ab)\rho.$$

Also for the equilibrium of the column of fluid passing through the disc of the screw, $T=\rho S(1+a)abV^2$, since the mean pressure over the boundary is equal to p.

Hence $\frac{\rho SV^2ab}{2}(2+ab)=\rho S(1+a)abV^2$, that is, $b=2$. Accordingly, then, with these ideal conditions one-half of the additional stemward velocity is given to the fluid at the front of the screw.

The above result may be obtained in a slightly different manner. Assuming Bernouilli's Equation to apply to the inflowing and outflowing streams, the pressure immediately in front of the screw

$$=p+\frac{\rho V^2}{2}-(1+a)^2\frac{\rho V^2}{2},$$

and the pressure immediately at the back of the screw

$$=p+\frac{\rho V^2}{2}(1+ab)^2-\frac{\rho V^2}{2}(1+a)^2.$$

Hence the thrust T of the screw

$$=\frac{\rho}{2}SV^2\left[(1+ab)^2-1\right].$$

Also since the mean pressure over the boundary is equal to p,

$$T=SV^2ab(1+a)\rho.$$

Equating the two values for the thrust we have as before, $b=2$.

In the preceding discussion it is assumed that the screw is rotating at a

* This nomenclature differs from that of the Royal Aëronautical Society, which suggests that $(1+b)V$ should represent the velocity relative to the screw of the outflowing column of fluid.

stationary point in a fluid which has an undisturbed velocity V and pressure p. Exactly the same result would be obtained, of course, if we assume the screw to be moving forward with a velocity V in a fluid which at a sufficient distance from the screw, has a pressure p and no motion.

If aV = the inflowing velocity of the fluid at the disc of the screw, measured relative to the undisturbed fluid,

and abV = the outflowing velocity of the fluid at the disc of the screw, measured relative to the undisturbed fluid,

then $(1+a)V$, and $(1+ab)V$ represent the same velocities as before.

Hence $T=SV^2ab(1+a)\rho$.

Also kinetic energy of translation carried away by the fluid $=\dfrac{(1+a)VS\rho.a^2b^2V^2}{2}$

$$=\frac{abV.T}{2}$$

Also useful work done $=VT$.

The total work done by the screw on the fluid $=T(1+a)V$.

Hence $(1+a)VT=\dfrac{abVT}{2}+VT$, so that $b=2$.

$$\text{The efficiency of working}=\frac{\text{Useful work}}{\text{Total work}}.$$

$$=\frac{VT}{TV(1+a)}=\frac{1}{(1+a)}.$$

The Froude Theory applied to an Airscrew working in a Wind Tunnel

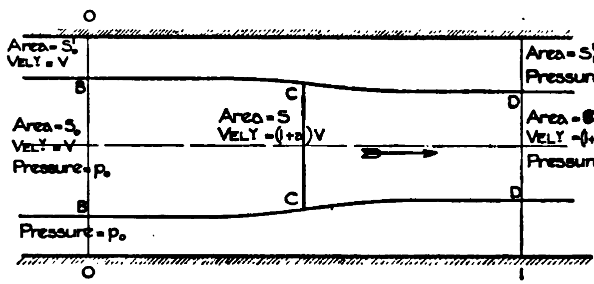
An interesting application of the Froude Theory may be made for the case of an airscrew working in a wind tunnel. As far as the writer is aware this case was first considered by Mr. McKinnon Wood of the Royal Aircraft Establishment. The present method of treating the problem is, however, somewhat more illustrative than that of Mr. Wood, who did not consider the forces and pressures acting at the boundary of the stream of air passing through the airscrew. A diagrammatic representation of the flow is shown in Fig. 12. Both the velocity and the pressure are considered to be uniform across the datum section 00 of the tunnel. At the section 1 the pressure is uniform ; also the velocities of the outflowing stream and the surrounding stream, although uniform, are of different magnitude. The motions of the inflowing stream and also of the stream surrounding both the inflowing and the outflowing columns of air are such that at any point the sum of the velocity head and the pressure head has the constant value of $\left(\dfrac{\rho V^2}{2}+p\right)$.

With the outflowing stream the total head at any point is equal to $\dfrac{\rho}{2}[V+abV]^2+p_1$. With the preceding assumptions it will be apparent that the stability at the boundary BCD of the air flowing through the airscrew needs consideration.

Assume a tube of longitudinal section BCD and of infinitesimal thickness to separate the air flowing through the airscrew disc from the surrounding air.

Let F_0=the force exerted by the outside of the tube on the surrounding air, and F_1=the force exerted by the inside of the tube on the contained air.

Forces are regarded as positive when acting in the general direction of the air-flow. The symbols used in the investigation are given in Fig. 12.



$S_0 + S'_0 = S_1 + S'_1$ = area of cross section of channel.

FIG. 12.

Considering the equilibrium of the column of air without the tube it follows since viscosity is ignored that

$$\rho S'_1\left[V-\frac{S_1}{S'_1}abV\right]\frac{S_1}{S'_1}abV+S'_0p_0-S'_1p_1+F_0=0 \quad \ldots\ldots\ (1)$$

For the equilibrium of the air within the tube

$$-S_1(1+ab)V\rho.abV+S_0p_0-S_1p_1+F_1+T=0 \quad \ldots\ldots\ (2)$$

Whence adding we get

$$-(abV)^2\rho S_1\left[1+\frac{S_1}{S'_1}\right]+\left[S_0+S'_0\right]\left[p_0-p_1\right]+T+F_0+F_1=0.$$

Also since the total head of the surrounding air is constant

$$(p_1-p_0)=\frac{\rho V^2}{2}\cdot\frac{abS_1}{S'_1}\left[2-\frac{abS_1}{S'_1}\right].$$

Assuming the total force on the tube to be zero it follows that

$$T=(abV)^2\rho S_1\left[1+\frac{S_1}{S'_1}\right]+\frac{1}{2}\rho V^2abS_1\left[1+\frac{S_1}{S'_1}\right]\left[2-\frac{abS_1}{S'_1}\right] \quad \ldots\ (3)$$

The increase of head at the airscrew disc

$$=\frac{T}{S}=(p_1-p_0)+\frac{1}{2}\rho V^2ab[2+ab].$$

Whence $$T=\frac{\rho SV^2}{2}\left[ab(2+ab)+\frac{abS_1}{S'_1}\left(2-\frac{abS_1}{S'_1}\right)\right] \quad \ldots\ldots\ (4)$$

When T, V, S, and the cross-sectional area $(S_1+S'_1)$ of the channel are known the values of S_1, S'_1, and ab may be calculated from the expressions (3) and (4).

The forces acting on the outside and on the inside of the tube BCD may be calculated from the expressions (1) and (2). Although the total force acting on the tube is zero it does not follow with this hypothetical flow that the tube is inoperative. It can easily be shown that if the tube is severed at any section equal and opposite forces—in some cases of appreciable magnitude—will be acting on the two parts. If for any values of T, V, S, p_0, and $(S_1+S'_1)$ the values of b and p_1 are calculated it will be found that b is slightly less than 2 and also that (p_1-p_0) has a small positive value.

A Mathematical Investigation of the Nature of the Air-flow around an Airscrew

An interesting theoretical investigation of the nature of the air-flow around an airscrew has been made by Cowley and Levy.* As with the Froude Theory the air is assumed to be both inviscid and incompressible. Also all rotational motions are neglected. For convenience the problem considered is that of an airscrew rotating at a stationary point in a current of air which at a sufficient distance from the airscrew has a uniform velocity V and a uniform pressure p_0. Due to the work done by the thrust of the airscrew, the total head of the air which has passed through the airscrew disc is greater than that of the surrounding air. In both cases the flow is assumed to be such that Bernoulli's Equation is satisfied. Thus the motion of the air surrounding the outflowing stream from the airscrew, is such that the total head, which in this case is the sum of the velocity head and pressure head, has the constant value of $\left(\frac{\rho V^2}{2}+p_0\right)$. At any point within the outflowing stream the total head is also constant at a value of $\left(\frac{\rho V^2}{2}+p_0+H_0\right)$ where H_0 is the augmented head—due essentially to the thrust—in the immediate vicinity of the point.

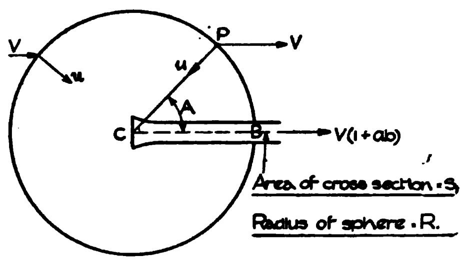


Fig. 13.

A diagrammatic representation of the flow postulated by these writers is shown in Fig. 13. The air flowing through the airscrew disc is assumed to be drawn with a uniform radial velocity through the surface of a large sphere of radius R and of centre C ; the area S_1 of the sphere through which the air is discharged is here excluded. With steady conditions of flow, the mass of air carried away by the outflowing stream is equal to the mass of air flowing through the remaining surface of the sphere. As with the Froude theory there is a surface discontinuity of velocity between the outflowing stream and the surrounding air. The thrust of the airscrew is found by considering the equilibrium of the sphere, when

* "Aeronautics—In Theory and Experiment." Published by Arnold & Co.

the motion of the air, both within and without, is steady. Obviously in this case the total force acting on the sphere, which is equal to the sum of the thrust of the airscrew and the integral of the resolved pressures on the boundary, is equal to the rate of transference of momentum from within to without the sphere. For simplicity it will be assumed that the velocity of the air in the outflowing stream is uniform and of magnitude $(1+ab)V$.

Also if u represent the magnitude of the uniform radial velocity across the surface of the sphere, and S_1 the cross-sectional area of the outflowing stream, the mass of air flowing into the sphere in unit time is equal to $(4\pi R^2 - S_1)u\rho + S_1\rho V$, that is $(4\pi R^2 u\rho + S_1\rho V)$, since if R be very large u is very small.

Also the mass of air carried away in unit time by the outflowing stream is $S_1\rho V(1+ab)$.

Hence $u = \dfrac{S_1 V.ab}{4\pi R^2}$.

The velocity at any point P on the surface of the sphere is

$$\sqrt{(V^2+u^2-2Vu\cos A)},$$

so that the pressure normal to the sphere

$$= p + \frac{\rho}{2}[V^2 - (V^2+u^2-2Vu\cos A)]$$

$$= (p+\rho V.u.\cos A) \text{ approximately.}$$

The component of this pressure in the direction CB

$$= -(p+\rho Vu\cos A)\cos A.$$

Hence the force acting on the sphere which is due to the pressure components in the direction CB

$$= -\int_0^\pi 2\pi R^2 \sin A(p+\rho Vu\cos A)\cos A\, dA$$

$$= -\frac{4\pi R^2\rho Vu}{3} = \frac{-\rho V^2 abS_1}{3}.$$

Hence the force acting on the system in the direction $CB = \left(\dfrac{-\rho V^2 abS_1}{3} + T\right)$.

This force must be equal to the rate of change of momentum of the system.

The mass of air leaving in one second, an annulus of radius R sin A and of area $2\pi R.\sin A.RdA$ is

$$2\pi R.\sin A.R.dA\rho(V\cos A - u).$$

Hence the momentum transferred in one second in the direction $CB = +2\pi R^2.\sin A.dA.\rho[V\cos A - u]\,[V - u\cos A]$.

The integral of this expression for the whole sphere where A ranges from 0 to π is $-\left(\dfrac{4S_1\rho.V^2.ab}{3}\right)$. To this integral must be added $-S_1\rho[V-u]^2$, since we are now considering only the air which is flowing into the airscrew. The momentum transferred in one second by the outflowing stream in the direction $CB = +S_1\rho(1+ab)^2V^2$.

Hence for the whole system the total momentum transferred in one second in the direction CB

$$= -\frac{4S_1\rho V^2ab}{3} - S_1\rho[V-u]^2 + S_1\rho(1+ab)^2V^2$$

$$= -\frac{4S_1\rho V^2ab}{3} - S_1\rho V^2 + S_1\rho V^2[1+ab]^2$$

$$= +S_1\rho V^2ab[2/3+ab].$$

But the total transference of momentum is equal to the force acting on the system.

Hence $-\frac{\rho V^2abS_1}{3} + T = S_1\rho V^2ab[2/3+ab]$,

so that $T = S_1\rho V^2ab[1+ab] = S(1+a)\rho.V^2.ab$,
where S=area of the airscrew and $(1+a)V$ is the inflow velocity measured immediately in front of and relative to the airscrew.

It will be noticed that this is the same expression for the thrust as was obtained from the Froude theory.

The de Bothezat Blade-Screw Theory

No attempt is here made to describe in detail the blade-screw theory developed by de Bothezat. The theory, which has been published in a Memoir by the Advisory Committee for Aeronautics, U.S.A., gives a mathematical analysis of the phenomena of the working of a screw and at the same time unites in a continuous whole the several states of work conceivable for a screw. The underlying basis of the theory is the employment of an empirical-theoretical method of solving the problem which is defined initially by the normal working conditions of the screw. Briefly stated this empirical-theoretical conception of the screw problem considers the fluid surrounding the screw to be subdivided into three regions. The first region includes the whole of the inflowing stream up to the cross-section where the local phenomena due to the rotation of the screw just begin to appear. The second region, which contains the screw, is immediately behind the first, and encloses that part of the stream disturbed locally by the rotation of the blades. The extent of this region is limited by the condition that the differences of pressure on the two extreme cross-sections is equal to the thrust of the screw. The third region is the direct prolongation of the second and extends up to the narrowest cross-section of the outflowing stream. Bothezat considers the fluid flow in the first and third regions to obey directly the laws of a perfect fluid, and also that the quantitative effect of the phenomena which occur within the second region can be estimated directly from experimental data.

With this theory a system of fundamental equations defining the working phenomena of the screw are established from the ordinary theorems of momentum and moments of momentum, in such a manner that the additional fluid velocities —both translational and rotational—which are due to the working of the airscrew, may be calculated and the whole fluid flow visualised. Also a fundamental theorem measuring the working losses of the screw is established. The theory is

sufficiently comprehensive to allow a general discussion of the sixteen states of work possible with a screw. With the ordinary propulsive screw, whether rotating at a stationary point or with a forward motion, the energy losses are divided into three classes : (*a*) a loss in ventilation, (*b*) a vortex loss, and (*c*) a resistance loss. The two fundamental parameters of the theory are, firstly *the relative pitch*, which is the pitch of the trajectory of a blade element measured by taking the pitch of the blade element as unity, and secondly *the specific function*, which is the ratio of the work of the thrust to the work of the torque.

The theory enables the most favourable working conditions of a blade element to be established. The effect on the performance of a screw of parameters such as the size, the shape, and the number of blades, and also the shape of the blade section is also considered. Using the calculus of variations an investigation is made of the best contour of an airscrew blade. To consider the question of the choice and adaptation of screws the notion of uniform families of screws which are aerodynamically alike is introduced. Screws of the same family are considered to be working at conditions of aerodynamic similarity when homologous sections of the blades of the several screws are geometrically alike and are working at the same angle of incidence. With this theory the influence of the rotational speed on the efficiency and the dimensions of a screw is examined in outline. Finally, Bothezat shows that screws may be divided into three classes, namely, major screws, optima or maxima screws, and minor screws, which are essentially different in their general properties.

Lanchester's Vortex Theory of an Airscrew

Lanchester has shown that the working of an airscrew, and also the attendant phenomena, may be studied and partly explained by a consideration of the

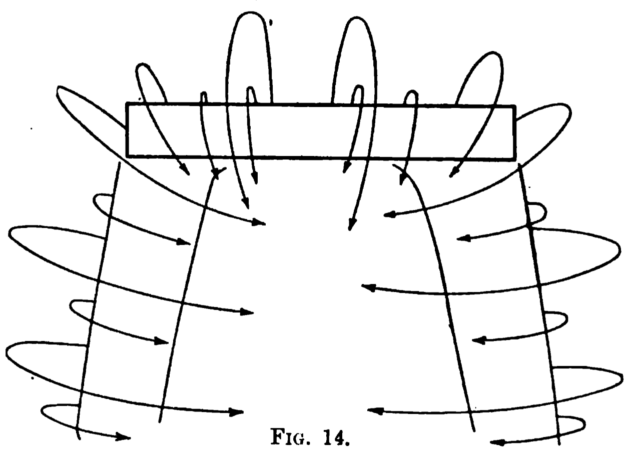


Fig. 14.

vortex motions created by the movement of the airscrew blades through the surrounding air. Before presenting a brief outline of this theory* it is proposed

* This vortex theory of an airscrew is published in "The Proceedings of the Automobile Engineers," Vol. IX.

to consider from the standpoint of the vortex theory the air-flow around an aerofoil. It is well known that with an ordinary aerofoil, the positive pressure on the lower surface and the negative pressure on the upper surface have maximum values at the central region; also that the values of both these pressures diminish towards the tip of the aerofoil. It follows then that the air passing beneath the aerofoil has in addition to a downward acceleration an outward acceleration towards one or other of the aerofoil tips, whilst the air passing above has an inward acceleration from the aerofoil tips to the central region of maximum negative pressure. When, therefore, these two neighbouring layers of air, the one from the upper surface and the other from the lower surface, meet at the trailing edge of the aerofoil, they have opposite motions along the trailing edge, that is, a vortex motion is created. The disturbance behind an aerofoil consists, therefore, of two equal

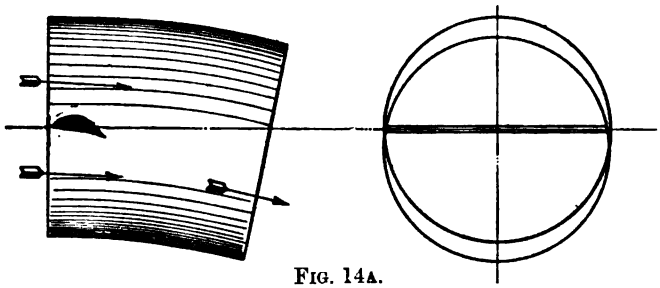


FIG. 14A.

and opposite vortices having a general downward motion. In fact the sustentation of the aerofoil in flight is a direct consequence of the continuous production of such a vortex pair. The two trailing vortices from the extremities are connected, as shown in the diagrammatic sketch* of Fig. 14, by a cyclic component of equal strength in the motion of the air surrounding the aerofoil.

In the present theory of an airscrew, Lanchester adopts a theoretical conception of the nature of the motion of the air in the neighbourhood of an airscrew, this conception being based essentially on the vortex theory of the dynamic support of an aerofoil, which has been just outlined. The residuary vortex motion in the wake of an airscrew blade has been represented dynamically by a cylindrical column of air of which the diameter is equal to the effective length of the blade. A graphical representation of this idea is given in Fig. 14A, which shows a blade of aerofoil shape arranged diametrically across the mouth of a slightly bent column. The capacity of this hypothetical circular column is a measure of the quantity of air controlled by the airscrew blade working in its spiral path. It is apparent that the adoption of a column of circular cross-section presumes definite pressure distributions over the surfaces of the blade. For the purpose of illustrating the theory Lanchester takes the effective blade length to be equal to three-fourths of the airscrew radius; although he suggests that

* The sketches of Figs. 14 and 14A have been copied from the paper to which reference has been previously made.

the best value will probably be found from experience. To apply the theory to practice both the efficiency and the resistance coefficients of the blade sections need to be estimated from either theory or experimental aerodynamical data. Moreover, it is necessary to estimate the magnitude of the inflow velocity. Lanchester assumes, on the basis of the Froude theory, that one-half of the velocity of the outflowing stream is given to the air at the front of the airscrew. In this paper, applications of the theory to a general treatment of the screw problem and also to the particular cases of the airscrew rotating at a stationary point, and the airscrew of highest efficiency, are considered in detail. The dependence of airscrew performance on parameters such as camber of blade sections, shape of plan form of the blade, and also the magnitude of the inflow velocity are also investigated. It is shown that the shape of the blade should be determined partly from a consideration of the surrounding vortex motions. As with an ordinary aerofoil, the blade shape may be regarded from the standpoints, firstly of the primary camber, which is related to the two-dimensional flow, and secondly from the secondary camber, which superposed on the primary camber is largely concerned with the cyclic motion around the blade itself.

CHAPTER III

STUDY OF THE AIR-FLOW AROUND AN AIRSCREW

General Discussion

From the momentum and aerofoil theories of an airscrew we have been able to study more or less superficially the nature of the air-flow around an airscrew. We know that the surrounding air is disturbed in such a manner that the forward velocity of a blade section, measured relative to the air immediately in front, is greater than that relative to the undisturbed air, and that this increase of velocity depends on the amount of work which the element is doing and its efficiency of working. Also, at any point of the inflowing column the motion of the air is such that the sum of the velocity and the pressure heads has a constant magnitude, which is equal to that of the undisturbed air.

It should here be mentioned that some of the air passing through the airscrew disc does not come from the front, but is drawn in at the blade tips from behind. Also with the inflowing stream the radial velocity would appear to be of some magnitude, although the spin is negligible. The air, when in the immediate neighbourhood of the airscrew, receives an increase of total head and is then discharged in a column, of which the sectional area at first contracts rather rapidly and then slowly increases until the diameter is greater than that of the airscrew. The boundary of the outflowing stream, which cannot be regarded as a surface but rather as a sleeve of eddying motions, is at first very sharply defined, but afterwards becomes less pronounced. The axial velocity across any section of the outflowing stream is not uniform, but increases from the boundary to a maximum value at some distance inwards and then gradually diminishes to a zero or a negative value in the neighbourhood of the centre of the column. The discharged air has in addition to this translational motion a general rotational motion in the same direction as that of the airscrew and also the inevitable irregular motions or eddies. With a large thrust the vortex motion in the periphery of the blade tips is very pronounced. Owing to the intermittent action of the blades both the translational and rotational velocities with which the air is discharged are not uniform but of a pulsating nature. It is to be expected that the structure of the outflowing stream, in so far as the air discharged from a blade element intermingles with that of the neighbouring annuli columns, is very analogous to the twisted strands of a rope.

The kinetic energy of the air in the discharged stream is a measure of the rate at which energy is being dissipated; so that it follows directly from the Froude theory that a given thrust may be most advantageously developed when a large mass of air is discharged at a low mean velocity.

Thus, assuming the airscrew to be moving into still air the thrust T may be

represented by Σmv_0, where m represents an elemental mass of air in the discharged stream which is moving with an axial velocity v_0. Also the kinetic energy of translation $=\Sigma\frac{1}{2}mv_0^2$.

If a given thrust is to be developed Σmv_0 will have a constant value, and in this case the kinetic energy of translation—which represents the greater part of the energy dissipated in the outflowing stream—will have a smaller value the greater the mass of air moved to develop the constant thrust.

It has been suggested by Riabouchinsky* that a greater airscrew efficiency would *probably* be obtained if the velocity of the discharged air were more uniform and also if the pulsations of velocity at any one point were reduced.

Several investigators have measured the distribution of velocity of the air surrounding an airscrew, although up to the present it would appear that the distribution of pressure has not been considered. An investigation is now in progress at the National Physical Laboratory to measure the distributions of both pressure and velocity.† It is, however, somewhat difficult to measure with good accuracy either the pressure or the velocity, since the region surrounding an airscrew is one where the speed, direction, and static pressure of the air varies from point to point.

A Method of Predicting the Thrust of an Airscrew from Observations of Total Head

Some experiments‡ have been made by Stanton and Marshall to estimate firstly the performance of each blade element, and secondly the total thrust, from the measurements in the immediate neighbourhood of the airscrew of the distributions of total head in both the inflowing and outflowing streams. It is pointed out by Stanton that with the inflowing stream there is no reason to suppose any appreciable departure from the hypothetical condition of the Froude conception, namely constancy of total head. With the outflowing stream, however, the conditions for streamline flow—it is here assumed that a streamline flow is one in which the sum of the velocity and pressure heads is constant—are somewhat different, since with any annular stream the magnitude of the increase of the total head depends on the position of the blade element. It follows then that at a cross-section immediately behind the airscrew the air is discharged with differences of static pressure, so that radial currents are set up. The distributions of total head—that is $(\frac{1}{2}\rho U^2+p)$, where p and U are the pressure and velocity respectively at any point—were made with a combination of a Pitot tube and yawmeter so that the axis of the Pitot tube could be pointed in the direction of the relative wind. The datum total head was measured at a point at a sufficient distance in front of the airscrew to be without the disturbance due to the working of the airscrew. The observations of total head made at a considerable number

* "Recherches sur les hélices Aériennes," Riabouchinsky, D. "Bulletin de l'Institut Aerodynamique de Koutchino," Fasicule II, 1909.

† Since going to the Press, this investigation has been completed.

‡ "On a method of estimating, from observations on the slip-stream of an airscrew, the performance of the elements of the blades and the total thrust of the screw," by T. E. Stanton, D.SC., F.R.S., and Dorothy Marshall, B.SC. Advisory Committee for Aeronautics, 1918.

of points in front of the airscrew were found to be in very close agreement, and accordingly it was concluded that the total head of each of the inflowing streams was constant at a value which was equal to that of the surrounding air. At several arbitrarily fixed values of the forward and rotational speeds, measurements were made of the increase of the total head of the air in passing through the airscrew disc. From the large numbers of observations taken it was possible to determine the increase of total head for the whole area of the column affected by the airscrew blades. For the several values of $\left(\frac{V}{nD}\right)$ a series of thrust-grading curves were plotted, which when integrated enabled a comparison to be made with the total thrust as measured with a dynamometer. It was found that the agreements between the absolute coefficients of thrust as obtained from such integration, and the absolute coefficients calculated from the thrust as measured directly, were remarkably good, especially at low values of $\left(\frac{V}{nD}\right)$. It is pointed out by Stanton that equality is not to be expected since the increase of total head as measured is smaller than that communicated by the airscrew because of blade friction and eddy motions. A direct application of this method to predict, from observations of the changes of total head of the air in passing through the airscrew disc, the thrust of an airscrew in flight, has been successfully used.

An Experimental Determination of the Velocity Distribution Around an Airscrew

An experimental investigation of the distribution of velocity in both the inflowing and outflowing streams of a model airscrew of diameter 16 in. has been made at the National Physical Laboratory by Pannell and Jones.* One of the objects of the investigation was to determine how the inflowing stream from the airscrew affected the forces on the rudder of an airship when turning. Accordingly some observations of velocity in a cross-section of the outflowing stream situated 8D behind the disc of the airscrew were made when the airscrew axis was inclined to the direction of the general wind. Measurements of the velocity distribution were also made at cross-sections of the outflowing stream situated at distances of 8D, 4D, 0·5D, 0·19D, and 0·06D behind the airscrew disc, where D represents the diameter of the airscrew. With the inflowing stream the distributions of velocity were measured at two cross-sections, situated 0·25D and 0·125D in front of the airscrew. The velocity observations were taken with a yawmeter so that both the magnitude and the direction of the resultant velocity at any point were measured. The data of the experiments are presented in terms of the axial, the radial, and the tangential components of the resultant velocity. The velocity distributions at the several sections were made with the airscrew rotating at a stationary point and at several values of $\left(\frac{V}{nD}\right)$ corresponding to the working conditions of practice.

* "An investigation of the nature of the flow in the neighbourhood of an airscrew," by J. R. Pannell, A.M.I.M.E., and R. Jones, M.A. Advisory Committee for Aeronautics, 1917.

It was found that at these values of $\left(\frac{V}{nD}\right)$ the minimum diameter of the inflowing stream was less than 0·75D, and would appear to be situated at an axial distance of between 0·2D and 0·5D behind the airscrew disc. The boundary of the inflowing stream was very sharply defined within an axial distance of 0·5D, but at greater distances became less pronounced. From the section of minimum diameter the outflowing stream was found to diverge, the diameter of a cross-section eventually becoming greater than that of the airscrew. The boss of the airscrew had an appreciable retarding influence on the local velocity. It was found that the spin, which was inappreciable on the inflow side, was very pronounced in the outflowing stream, even at a distance of 8D behind the airscrew. As would be expected, the radial velocity in front of the airscrew was quite appreciable.

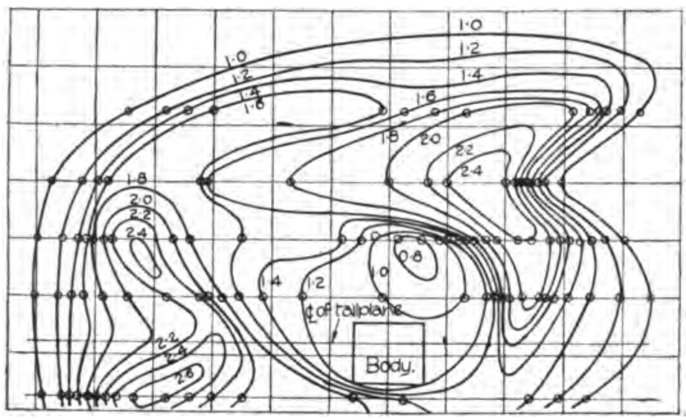
Influence of an Aeroplane on the Velocity Distribution around an Airscrew

A study of the velocity distribution around an airscrew when working at the conditions of practice is of some importance. With the present method of design, some part of the tail is within the outflowing stream of the airscrew, so that both the controllability and the stability of an aeroplane depend on the characteristics of the airscrew. Several experiments have been made at the Royal Aircraft Establishment with aeroplanes in flight to measure the velocity distribution in the outflowing stream of the airscrew. We shall first describe the investigation* of the velocity distribution in the outflowing stream of the tractor airscrew of the B.E. 2c aeroplane. In this case the velocity distributions at several forward speeds and at several values of the forward advance per revolution were measured over a vertical plane situated about 33 in. in front of the leading edge of the tail plane. If V be the speed of the aeroplane and abV the augmented velocity at any point of the outflowing stream—both velocities being measured relative to undisturbed air—the value of $\frac{(V+abV)^2}{V^2}$ may be conveniently expressed by $\left(\frac{r}{r_0}\right)$. The ratio $\left(\frac{r}{r_0}\right)$ may be regarded as a resistance factor, from which the increase of resistance due to the augmented velocity of the outflowing stream may be calculated. From conditions of similarity it would be expected that the value of $\left(\frac{r}{r_0}\right)$ would be a function of the forward advance per revolution only. It was found, however, that at some points of the outflowing stream $\left(\frac{r}{r_0}\right)$ was a function not only of $\left(\frac{V}{n}\right)$ but also of V. Contour lines showing the lines of

* "Exploration of the air speed in the airscrew slip-stream of a tractor machine." Presented by the Superintendent, Royal Aircraft Establishment. Advisory Committee for Aeronautics, 1917.

constant velocity in the vertical cross-section of the outflowing stream were drawn for several forward speeds and several advances per revolution ; a representative contour map is shown in Fig. 15. In this case the airscrew has a forward advance of 6·4 ft. per revolution and a forward speed of 50 miles per hour. Each contour map has several pronounced characteristics. Looking in the direction of the forward motion from behind the vertical plane over which the velocity distributions were measured, the velocity of the outflowing stream had maximum values at the top-right and bottom-left corners. These points of maximum intensity of

Velocity Contours of Outflowing Stream.



Forward speed of airscrew - 50 m.p.h.
Rotational speed of airscrew - 690 r.p.m.
The figures on the contours represent values of $\left(\frac{F}{F_0}\right)$.

Fig. 15.

velocity vary in position with the forward speed. Generally speaking the slower the forward speed the greater is the angular displacement in the direction of rotation of the airscrew. It would seem that the outflowing stream after leaving the airscrew is divided by the body and the wings into two distinct streams, the one on the left flowing slightly downwards and the other on the right slightly upwards in consequence of the counter-clockwise rotation of the airscrew. In all probability there is a third stream passing underneath the lower wing. In the present experiments, however, no attempt was made to establish the existence of such a stream. The region in the middle of the outflowing stream where the velocity is a minimum is lower relative to the tail plane, the greater the forward speed of the aeroplane. This would be expected, since the angle of incidence of the wings is smaller at the higher speeds—so that the outflowing stream will be deflected downwards by the surrounding air—although this deflection is somewhat lessened by the greater downwash at the lower forward speed. There are, therefore, two principal effects which are functions of the air speed only :

(*a*) The deflection of the outflowing stream, at a constant advance per revolution, due to changes of the angle of incidence of the wings and a varying forward speed of advance ; the effect is small where the velocity contours are flat, and (*b*) the variation of the drag of the body and the wings with the forward speed : this would of course not affect those parts of the outflowing stream which are without the interference from the wings and the body.

The parts of the outflowing stream where $\left(\frac{r}{r_0}\right)$ would be independent of V and a function of $\left(\frac{V}{n}\right)$ only would therefore be the regions of maximum velocity at the top-right and bottom-left corners of the contour diagram.

It will be noticed that the region of high velocity at the bottom left-hand corner is divided by a line on a level with the leading edge of the tail plane, which is probably an indication of the retarding effect of the tail plane in a region situated 33 in. in front. These experiments show that with a tractor aeroplane the velocity of the outflowing stream in the neighbourhood of the tail plane has been considerably reduced by the drag of the body and the wings. This is of course not the case with the parts of the stream which have come through the spaces between the wings and the body. When calculating the lift on a tail plane of a tractor it is well to remember that the velocity in this neighbourhood is probably much lower than that immediately behind the airscrew disc. From the data of these experiments it was estimated that the average reduction of air speed with a tractor aeroplane is such that if the slowing-up of the outflowing stream due to the body and wings be ignored the area of the tail plane as calculated to give a definite lift is probably about 80 per cent of the area needed in practice.

Measurement of the Spin of the Outflowing Stream from an Airscrew

Some experiments* were made with a pusher aeroplane to measure in flight the spin and the position of the centre line of the outflowing stream from the airscrew.

The measuring instrument was a small double vane fitted to the kingpost on the front spar of the tail plane, the fin being removed and a balanced rudder fitted, to avoid interference with the vane. The height of the vane was adjusted within ranges of 2 ft. above and 2 ft. below the tail plane. From these experiments it would appear that the rotation of the outflowing stream appreciably affected the aerodynamic performance of the tail plane. Thus, with the fin and the rudder in the outflowing stream, it was necessary to use the rudder in order to fly straight at all speeds. The experiments show that perfect balance at all speeds can only be obtained when the tail plane is without the outflowing stream. It is to be expected, however, that an approximate balance would be obtained with the fin and rudder at the centre of the outflowing stream.

* "Experiments on the rotation of a propeller slip-stream in a pusher aeroplane." Presented by the Superintendent of the Royal Aircraft Factory. Advisory Committee for Aeronautics, 1917.

AN ANALYSIS OF THE ENERGY ACCOUNT OF AN AIRSCREW

We have already seen that of the total energy put into an airscrew a large proportion is converted into useful work, the greater part of the remainder being carried away in the outflowing stream, and also that the useful work done is a quantity which can be measured with good accuracy. There is, however, no exact quantitative knowledge of the various other forms of energy co-existing in the outflowing stream.

Any attempt, therefore, to analyse the energy account of an airscrew must, by virtue of the complexity of the problem, be of a somewhat speculative character. On the other hand, however, although the various forms of energy—kinetic, potential, heat, sound—are inextricably mingled, some by their nature are negligibly small, whilst of the others the magnitudes of those which have a direct connection with the forces of the airscrew can be calculated with good accuracy.

It is now proposed to consider in detail a paper* written by the author in 1918 in which is given a theoretical analysis of the energy account of an airscrew. It would seem that the total energy, E, put into an airscrew in unit time eventually reappears in one or other of the following forms :

(*a*) Useful energy, E_1. A quantity which can be readily calculated from the thrust, T, and forward speed, V. Thus $E_1 = \frac{VT}{550}$ H.P. if V is measured in feet per second and T in lb.

(*b*) Kinetic energy of translation E_2 ; a function of both the thrust and the mass of air flowing into the airscrew. When calculating the magnitude of E_2, we may assume, without a great sacrifice of accuracy, the hypothetical streamline flow of Froude. In such a case if δM be the mass of air flowing into an annulus of the airscrew disc of radius r and width dr, then

$$\delta M = \rho(1+a)V.2\pi r dr$$

and $$\delta T = \rho(1+a)V.2\pi r dr.abV = abV.\delta M,$$

where aV is the mean inflow velocity into the annulus and abV is the augmented velocity of the mass when in the outflowing stream.

Hence $$E_2 = \int_{r=0}^{r=\frac{D}{2}} \frac{(\delta M)(abV)^2}{2 \times 550} = \int_{r=0}^{r=\frac{D}{2}} \frac{(\delta T)(abV)}{1100}.$$

(*c*) Kinetic energy of rotation, E_3, a concomitant effect of the torque. Since the torque is a measure of the angular momentum of the air, it would follow that the air in the vicinity of the airscrew has a kinetic energy of rotation, E_3, where

$$E_3 = \int_{r=0}^{=\frac{D}{2}} \frac{1}{2} \cdot \left(\frac{\delta Q}{r.\delta M}\right)^2 \cdot \frac{\delta M}{550} = \int_{r=0}^{r=\frac{D}{2}} \frac{abV(\delta Q)^2}{r^2(\delta T).1100}$$

* "An analysis of the energy account of an airscrew, with an application to the case of the tandem airscrew." With Appendix. By A. Fage, A.R.C.SC., D.I.C. Advis. Comm. Aeron., 1918.

This kinetic energy, E_3, is due to a regular angular rotation about the axis of the airscrew, although such motion is somewhat hidden by the various irregular motions of the outflowing stream.

(*d*) The remaining energy, E_4, most of which is in the outflowing stream. E_4 represents a collection of energy of various forms, but chiefly the energy of confused motion. An attempt to subdivide E_4 into its constituent parts is now made. It should, however, be fully realised that no accurate calculation of the magnitude of each of these forms of energy is possible.

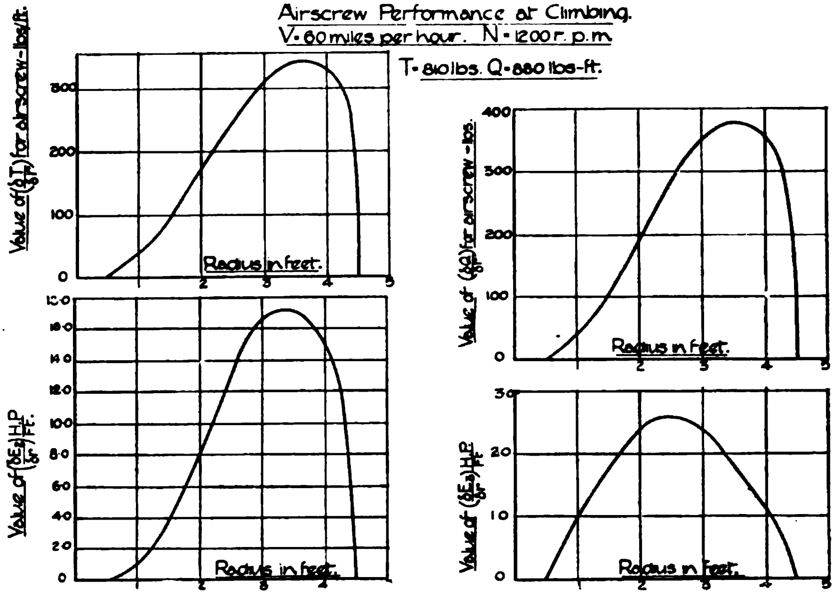


FIG. 16.

(E_4a). Energy dissipated as sound waves. Magnitude not calculable, but probably small.

(E_4b). Mechanical losses. One form of mechanical loss would be the vibrational working of the airscrew.

(E_4c). Frictional and heat losses, including the energy of irregular eddying motions, at the tip and at the surface of discontinuity.

(E_4d). Kinetic energy of translation due to *irregular* linear motions.

(E_4e). Pressure energy.

(E_4f). Energy of any regular vortex motion.

Of the various forms of energy collected under the heading "E_4," the only energy of reconvertible form is that of (E_4f), that is the energy of the vortex motion, of which the direction of rotation depends on that of the airscrew.

With the object of finding the relative magnitudes of E, E_1, E_2, E_3, and E_4, calculations of the energy account are made for a fairly representative airscrew. The airscrew chosen has a diameter of 9 ft., and is driven through a reduction gearing of $\frac{35}{59}$ by a 210 h.p. Hispano-Suiza engine. Calculations of the

energy accounts are made for the two principal working conditions of the airscrew, firstly at the maximum horizontal flight speed of 100 m.p.h., at which the airscrew is assumed to have a rotational speed of 1250 r.p.m., a thrust of 575 lb., and a torque of 860 lb.-ft.; and secondly at a climbing speed of 60 m.p.h. and a rotational speed of 1200 r.p.m., the corresponding values of the thrust and torque being 810 lb. and 880 lb.-ft. respectively. The data for the climbing speed are given in detail in Table IV, and shown graphically in Fig. 16. The calculations were made according to the preceding theory, the assumed value of " b " being 2. The energy accounts for both conditions of working—climbing and maximum horizontal flight speed—are collected under the heading " Theoretical data " in Table V.

With the object of substantiating the general conclusions of these theoretical calculations, the data of calculations based on the experimental measurement* of the velocity distribution around an airscrew are given in column 3 of Table V. In this case the energy of translation due to the axial motion, and the energy of rotation at a section of the outflowing stream situated about one diameter behind the disc of the airscrew were calculated from the measured velocities of translation and rotation for the climbing speeds of the airscrew.

We have already seen that the efficiency of an airscrew as calculated from the assumptions of Froude is $\left(\frac{E_1}{E_1+E_2}\right)$. The efficiency in practice is $\left(\frac{E_1}{E}\right)$, so that the ratio of the practical and " Froude " efficiencies is $\left(\frac{E_1+E_2}{E}\right)$, that is about 0·82 for both climbing and horizontal flight (see Table V). The energy of rotation, E_3, due to the torque is only about 3 per cent of the total energy put into the airscrew. Moreover, the value of (E_3/E) decreases with an increase of (V/nD). The energy losses which are not directly calculable, E_4, are about 15 per cent of the total energy, E. As the working conditions change from those of the maximum horizontal flight to those of climbing, the values of both E_2 and E_3—the translational and rotational energies respectively—increase.

Reconvertible Energy of the Outflowing Stream

The present analysis of the energy account of an airscrew has a direct bearing on the question whether it is possible to improve the efficiency of a tandem combination by rotating the two airscrews in opposite directions, the rear airscrew taking from the outflowing stream of the front airscrew some of the rotational energy. From the analysis it is shown that the energy in the outflowing stream which is due to more or less regular rotational motion is of two forms, (*a*) kinetic energy (E_3) due to a regular rotational motion about the axis of the airscrew, and (*b*) the energy of any regular vortex motion (E_{4g}). Viscosity degrades both in character and magnitude these two rotational motions, the breakdown increasing with the distance behind the airscrew. At the region of the outflowing stream of the front airscrew in which the rear airscrew of a tandem combination

* These calculations were made from experimental data of the paper described on p. 33.

works, it is to be expected that the kinetic energy of rotation as calculated from the torque will not be greater than 3 per cent of the total energy put into the front airscrew, so that *assuming the whole of this energy* to be absorbed or balanced by the rotational energy of the outflowing stream of the rear airscrew, the resulting increase of the efficiency of the combination would be greater by about 3 per cent than that of a combination in which the airscrews rotate in the same direction. The magnitude of the energy of any regular vortex motion (E_4f) of the outflowing stream of the front airscrew, at the vicinity of the rear airscrew, cannot be calculated with any accuracy. It is, however, expected not to be large, since of the 15 per cent of the total energy which is dissipated in forms such as sound, heat, frictional, mechanical, pressure, eddying motion, vortex motion, the larger part is probably carried away by irregular eddy motion. Moreover, it is unlikely that the whole of the regular vortex and rotational motions from the front airscrew will be taken away from the outflowing stream by the superposition of equal and opposite motions from the rear airscrew. From this reasoning, then, it appears that the efficiency of a tandem combination may be increased by about 3 per cent if the airscrews rotate in opposite directions. It will be shown elsewhere that the general accuracy of this statement is supported by experiment.

TABLE IV

ANALYSIS OF THE ENERGY ACCOUNT OF AN AIRSCREW.
AIRSCREW PERFORMANCE AT CLIMBING

$V = 60$ m.p.h.
$N = 1200$ r.p.m.
$T = 810$ lb.
$Q = 880$ lb.-ft.

Diameter of the airscrew $= 9$ ft. Efficiency $= 64\cdot5$ per cent.

r (feet)	$\left(\frac{\delta T}{\delta r}\right)$ for airscrew (lbs./ft.)	$\left(\frac{\delta Q}{\delta r}\right)$ for airscrew (lbs.)	Value of "a."	$\left(\frac{\delta E_2}{\delta r}\right)$ h.p./ft.	$\left(\frac{\delta E_3}{\delta r}\right)$ hp./ft.
4·25	300·0	315·0	0·246	11·80	0·72
4·00	330·0	356·0	0·280	14·80	1·08
3·50	340·0	379·0	0·319	17·20	1·77
3·00	310·0	349·0	0·337	16·70	2·36
2·50	247·0	276·0	0·324	12·80	2·57
2·00	173·0	188·0	0·292	8·10	2·38
1·50	98·0	102·0	0·232	3·64	1·73
1·00	40·0	39·0	0·153	0·98	0·93

$E = 201$ h.p. $E_1 = 129\cdot5$ h.p. $E_2 = 37\cdot5$ h.p. $E_3 = 6\cdot5$ h.p. $E_4 = 27\cdot5$ h.p.

TABLE V

Energy Account of Airscrew

	Theoretical data.		Data calculated from the measured air-flow around an airscrew.
	Horizontal flight.	Climbing.	Climbing.
Energy put into airscrew . E	$205 \cdot 0 = 1 \cdot 00E$	$201 \cdot 0 = 1 \cdot 00E$	$1 \cdot 00E$
Useful energy . . E_1	$153 \cdot 4 = 0 \cdot 749E$	$129 \cdot 5 = 0 \cdot 645E$	$0 \cdot 67E$
Energy appearing as kinetic energy of translational in the outflowing stream . E_2	$14 \cdot 2 = 0 \cdot 069E$	$37 \cdot 5 = 0 \cdot 186E$	$0 \cdot 11E$
Energy appearing as kinetic energy of rotational in the outflowing stream . E_3	$4 \cdot 6 = 0 \cdot 022E$	$6 \cdot 5 = 0 \cdot 032E$	$0 \cdot 035E$
Other energy losses . E_4	$32 \cdot 8 = 0 \cdot 160E$	$27 \cdot 5 = 0 \cdot 137E$	$0 \cdot 185E$

CHAPTER IV

THE PRINCIPLE OF DYNAMICAL SIMILITUDE

A General Exposition

Before the value of experiments with model airscrews can be fully appreciated it is necessary to consider in some detail the law of dynamical similitude, which is the basis from which the performance of the full-scale airscrew is calculated from the experimental data of the model. Sir George Greenhill has pointed out that this law follows directly from a theorem in Newton's " Principia " (Book II, Prop. XXXII). Since that time, however, more complete demonstrations of this principle have been made by Reynolds, Lord Rayleigh, and numerous other experimenters who have established beyond doubt its great practical utility. The airscrew affords a most complete and interesting illustration of the applicability of the law of dynamical similitude. Before considering in some detail this problem it should be mentioned that this particular application of the law has been considered very exhaustively by both Bramwell* and Durand.† The author feels that he cannot do better than to develop the subject along the lines suggested by both these writers.

If it be assumed that there is a relationship between the forces acting on both the model and the full-scale airscrew it naturally follows that there must be a functional relationship between the forces acting on an airscrew and the various attendant determining conditions and dimensions. These various parameters, on which the performance of an airscrew depend, have been arranged by Durand in the following three groups :

(*a*) All those dimensions and characteristics necessary to define the airscrew as a geometrical and physical entity.

(*b*) All the mechanical or dynamic characteristics of the air in which the airscrew moves.

(*c*) The conditions of operation, which involve the velocity relationships between the airscrew and the air, and also between the airscrew and the earth.

Durand further points out that these characteristics or variables may be again subdivided into two classes. In the first class are grouped all those variables of which the values may be definitely expressed or measured, e.g. a geometrical dimension, a velocity, the density or the viscosity of the air. The remaining

* " Some notes on the effect of size on the efficiency and performance of airscrews," by F. H. Bramwell. Advis. Comm. Aeron., 1912.

† " A brief discussion of the law of similitude as affecting the relation between the results derived from model forms and those to be anticipated from full-sized forms," by W. F. Durand. Amer Advis. Comm. Aeron.

characteristics or parameters, which are such that for any one no numerical measure can be directly determined in any case, are collected in the second group. Illustrations of such characteristics are, turbulence in the air, roughness or quality of the surface of the airscrew, the influence of the form of the airscrew independent of the dimension.

The force reaction between the airscrew and the surrounding air may therefore be regarded as dependent on two functions—one of known or determinable form, and the other of unknown form. The latter function will represent a relationship between all the parameters which are a characteristic of the problem, no matter whether the magnitude of each parameter can be definitely assigned or not. This " unknown " function cannot be calculated algebraically for any specific case, but may be determined experimentally when the force acting on the airscrew is measured and account is also taken of the known parameters and also of the character of the known function. It is then the object of experiments with a model airscrew to determine under specific working conditions the magnitude of the " unknown " function. The performance of the full-scale airscrew when working under precisely similar dynamical conditions can then be calculated from this coefficient and the known parameters of the operating conditions. When applying the law to the case of the airscrew the assumption of complete geometrical similarity is made so that the airscrew may be defined by a single dimension and there is no need for a " form " parameter. In this connection it must be noted that complete geometrical similarity of form would necessarily imply geometrical similarity of the irregularities or roughness of the surface. Roughness of surface is, however, of minor importance, and it is perhaps unwise to venture too far into the abstract as several of the parameters which would need to be taken into consideration in a rigid application of the law of dynamical similarity are of very small importance in so far as the comparison of the results of model and full-scale experiments is concerned. The practice then is to consider only the parameters which are known to influence the forces acting on the body, the final justification of the initial assumptions made being obtained from experiment and experience rather than from abstract reasoning. The principal parameters on which performance of an airscrew depend are—

(*a*) Characteristics of the airscrew as a physical and geometrical body.
- (1) The diameter.
- (2) The blade angles at the several sections.
- (3) The plan form of each blade.
- (4) The area of each blade.
- (5) The area and form of each cross-section of the blade.
- (6) The form and size of the boss.
- (7) The smoothness of the blade surface.
- (8) The density of the material of the blade.
- (9) The elastic coefficients of the material of the blade.

(*b*) The characteristics of the air.
- (10) Density.
- (11) Viscosity:

(12) Compressibility. This is usually expressed by the velocity of sound, which is dependent on both the density and elasticity of the air.

(13) Character and extent of turbulence.

(c) The characteristics of operation.

(14) Speed of translation.

(15) Speed of rotation.

Assuming full geometrical similarity, a single controlling dimension, such as diameter, with the assumed geometrical form defines completely the airscrew, so that the characteristics (2), (3), (4), (5), (6) are redundant. Of the remaining characteristics (7) and (13) do not admit of numerical evaluation and are regarded as similar with both model and full-scale airscrews.

Adopting the usual notation we have :—

	Dimension.
Thrust, T	MLT^{-2}
Torque, Q	ML^2T^{-2}
Diameter, D	L
Rotational speed, n	T^{-1}
Translation speed, V	LT^{-1}
Density of air, ρ	ML^{-3}
Density of material of blade, Δ	ML^{-3}
Coefficient of elasticity of blade material, E . .	$ML^{-1}T^{-2}$
Kinematic viscosity of medium, ν	L^2T^{-1}
Velocity of sound in medium, $\overline{V}$	LT^{-1}

Employing the method of dimensional equations the general expressions for the thrust and the torque are—

$$T=\rho D^4n^2.f\left(\frac{V}{Dn},\frac{\nu}{D^2n},\frac{\overline{V}}{Dn},\frac{E}{D^2n^2\Delta},\frac{\Delta}{\rho}\right)$$

and $$Q=\rho D^5n^2.F\left(\frac{V}{Dn},\frac{\nu}{D^2n},\frac{\overline{V}}{Dn},\frac{E}{D^2n^2\Delta},\frac{\Delta}{\rho}\right)$$

These two expressions are rigidly correct, if the initial assumptions are correct, namely, that the thrust and torque depend only on V, D, n, $\overline{V}$, ν, E, ρ, and Δ. It naturally follows from the above expressions that to obtain dynamical similarity between two airscrews which are geometrically similar the following relationships must hold :—

(1) $\left(\frac{V}{Dn}\right)$ the same.

(2) $\left(\frac{\nu}{D^2n}\right)$ the same.

(3) Dn the same.

(4) $\left(\frac{E}{D^2n^2\Delta}\right)$ the same.

(5) $\frac{\Delta}{\rho}$ the same.

With the same working medium the only way in which the above expressions may be simultaneously fulfilled is by making D, V, n, E, and Δ respectively the same in each case, that is, the two airscrews are equal, and are working under the same conditions. Considering the physical meanings of the above five expressions, it is seen that relationship (1) implies that the forward advance per revolution expressed as a fraction of the diameter is the same with each airscrew. The second relationship shows that in order to obtain the same viscosity effects in the same medium, D^2n must be the same, that is, the rotational speed of a model as compared with that of the full-scale airscrew should be very high.

It follows from relationship (3) that compressibility of the air may be neglected if the airscrews have the same tip speed. If the airscrews are working at the same values of V and (nD) and if, further, the elastic constants and density of the material of the airscrew blades are the same, then from expressions (4) and (5) it is seen that the distortions will be the same.

It is now necessary to determine which of the above relationships can be readily satisfied with model experiments. Obviously the condition of (V/nD) constant can be most easily fulfilled, and it is then impracticable with model experiments to satisfy both the conditions (2) and (3) together.

When the conditions (1), (2), and (3) are satisfied together the model is equal to the full-scale airscrew. It will be noticed from conditions (4) and (5) that even if the elastic constants and density of the material of the blades were the same, the distortions of the model airscrew are smaller than those of the full scale, if the tip speed be smaller.

Neglecting the viscosity and compressibility of the air, and assuming whenever a comparison is made that the full-scale and model airscrews are geometrically similar, it follows that $T = \rho n^2 D^4 . f\left(\frac{V}{nD}\right)$

$$\text{and } Q = \rho n^2 D^5 . F\left(\frac{V}{nD}\right).$$

Hence with such assumptions the thrust of geometrically similar airscrews working at the same value of (V/nD) varies directly as the density of the air, the square of the rotational speed, and the fourth power of the diameter; also the torque varies directly as the density of the air, the square of the rotational speed, and the fifth power of the diameter. The efficiency is $\left[\dfrac{V.f\left(\frac{V}{nD}\right)}{2\pi nD.F\left(\frac{V}{nD}\right)}\right]$ and so has a constant value at any assigned value of (V/nD).

The applicability of the law of dynamical similitude of the above simple form in predicting the performance of full-scale airscrews from the data of model experiments depends largely on the magnitude of the errors which are introduced by neglecting the three factors—viscosity, compressibility, and distortion.

Often the model experiments are made at speeds appreciably lower than those at which it is desired to calculate the performance of the full-scale airscrew,

and it is then of primary importance to know what accuracy may be expected by assuming that at the same value of (V/nD) the thrust varies as ρn^2D^4 and the torque as ρn^2D^5. Experimental evidence, which is the deciding authority, shows that even with the omission to take account of these three factors the prediction of the performance of a full-scale airscrew from the data of model experiments can be made with moderately good accuracy in most cases.

It is necessary, however, to consider in more detail the influence of each of the factors of viscosity, distortion, and compressibility on the performance of an airscrew.

Viscosity Effect.

There is every reason to believe, so long as the scale of the model airscrew is not too small, that the viscosity of the air may be neglected. It is the practice at the National Physical Laboratory to test model airscrews at several translational speeds so that several values of the absolute coefficients of thrust, $(T/\rho n^2D^4)$, and of torque, $(Q/\rho n^2D^5)$, are obtained at the same value of (V/nD). With airscrews of one-third or one-fourth scale the values of these absolute coefficients, whether of thrust or of torque, are found to agree within the accuracy of the experiment. Further, with the model airscrew rotating at a stationary point $(V=0)$ it has been found experimentally that the thrust and the torque are each proportional to the square of the rotational speed so long as the value of the rotational speed is not too low. The author* has found, however, that the performance of a model airscrew of very small diameter is dependent on the viscosity of the air at low values of the translational and rotational speeds. Experiments with a model airscrew of diameter 18 in., which was made of aluminium to eliminate any distortion under load, showed that the "viscosity" effect was quite appreciable when the working speeds were sufficiently low. As would be expected, this effect has a very similar magnitude to that of an aerofoil of about the same size working at the corresponding values of both speed and angle of incidence. The effect of viscosity was most pronounced at the high values of (V/nD), when the angle of incidence of the blade sections is small. It would appear, then, that as far as the prediction of the performance of a full-scale airscrew from model experiments is concerned there is a lower limit of (DV) below which viscosity effects make an appearance.

Distortion of Airscrew Blades under Load.

It can easily be seen that if similar airscrews made of the same material are working at the same value of (V/nD) then the centrifugal force and the air loading of corresponding parts of each airscrew blade are proportional to (n^2D^4). It follows, then, that the compressive, tensile, and shearing stresses at corresponding parts of the blades of the similar airscrews are proportional to n^2D^2, that is, to the square of the tip speed. Assuming the density and the elastic constants of the material are the same, it follows, then, that at the same value of (V/nD) the strains, and therefore the distortions, at corresponding parts will be the

* "The 'Scale-Speed' effect on a model airscrew of small diameter," by A. Fage, A.R.C.SC., and H. E. Collins. Advis. Comm. Aeron., 1918.

same if the similar airscrews are running at the same tip speed. Unfortunately, it is impracticable in most laboratory experiments to run the model at the same tip speed as the full-scale airscrew, so that the distortions of these two airscrews at the same value of (V/nD) are probably slightly dissimilar. It is therefore of some importance, in view of the probable different amounts of distortion, to investigate experimentally the accuracy with which the results of the model experiments may be applied to practice. Experiments* made by M. Drzewiecki with a full-scale airscrew and a model of one-third scale show that when the measurements of thrust and of torque are made at the same values of V and of nD, the agreement between the two series of experiments was good. The accuracy of the general theorem that the strains of similar airscrews working at the same forward and tip speeds are the same is thus experimentally established. Eiffel† has found when similar airscrews are tested over a large range of speed the absolute coefficients of thrust and of torque are not constant at the same value of (V/nD), and he concludes that the deformations are different with the different air and centrifugal loadings. A comparison between the data of experiments made with models at the National Physical Laboratory and full-scale airscrews at the Royal Aircraft Establishment also shows that complete agreement cannot be expected. At the same time it should be mentioned that any conclusion based on a comparison of the data of model and full-scale experiments may be somewhat misleading, unless the accuracy of each set of experiments is definitely established.

In view of the completely unsymmetrical form of an airscrew blade it is perhaps not surprising that the strain due to both the air load and the centrifugal load should include torsion as well as bending. As far as the design of an airscrew is concerned it is of some importance that the deformation of the airscrew blades should be known, because the aerodynamic performance of an airscrew is determined from the shape of the blade under load. The subject of the deformation of airscrew blades under load has been treated experimentally and mathematically by Griffith‡ and Hague. They found experimentally that with airscrew blades as now made the deformations would appear to follow no definite law. Thus the out-of-balance twist for the principal working parts of two blades of an airscrew may be quite appreciable. In one experiment the angle of twist of two opposite blades of the same airscrew, when under the same load, were almost of equal magnitude, but of different sign, this anomalous result being probably due to differences of elastic coefficients of the material of the blades. Nevertheless, it is to be expected that a two-bladed airscrew is more likely to have " elastic " symmetry than a four-bladed one, assuming of course that the two opposite blades are shaped from the same laminæ. With a two-bladed airscrew, uniformity of physical properties of the same laminæ are only needed, whereas with the four-bladed airscrew uniformity of corresponding laminæ is needed in addition.

* " La résistance d'air et l'aviation," by G. Eiffel. " L'Aerophile," June, 1911.

† " Les nouvelles recherches expérimentales sur la résistance de l'air et l'aviation faites aux laboratoires du Champ de Mars et d'Auteuil," by G. Eiffel. " Mémoires de la Société des Ingénieurs Civils de France, 1912."

‡ " Preliminary report on the twisting of airscrew blades," by A. A. Griffith, M.ENG., and B. Hague, B.SC. Advis. Comm. Aeron., 1918.

Effect of Compressibility.

With model airscrews running at moderate tip speeds, compressibility of the air may be neglected without any appreciable sacrifice of accuracy. This, however, is not the case with modern full-scale airscrews where the tip speed often approaches the velocity of sound. There is at present no exact experimental evidence of what is occurring at the tip of a high-speed airscrew, where of course the compressibility effect is a maximum. Experiments are, however, in process at the National Physical Laboratory and the Royal Aircraft Establishment, with the object of measuring how the performance of an airscrew is affected by a high value of the tip speed. It is shown by Lord Rayleigh that a very good idea of the probable effect of compressibility may be obtained from the formula which connects the velocity and pressure of a gas expanding or contracting adiabatically. Imagine air which is at rest under a pressure p_0 to be set in motion so that its velocity becomes V and its pressure p.

Then the "adiabatic" equation is

$$\left(\frac{p_0}{p}\right)^{\frac{\gamma-1}{\gamma}}=1+\frac{(\gamma-1)}{2}\frac{V^2}{\bar{V}^2},$$

where γ is the ratio of specific heats and $\bar{V}$ is the velocity with which sound is propagated in the moving air.

Expanding the above expression we have

$$(p_0-p)=\frac{p\gamma V^2}{2\bar{V}^2}\left[1+\frac{1}{4}\left(\frac{V}{\bar{V}}\right)^2+\frac{(2-\gamma)}{24}\left(\frac{V}{\bar{V}}\right)^4+\ \ldots\ldots\right]$$
$$=\frac{\rho V^2}{2}\left[1+\frac{1}{4}\left(\frac{V}{\bar{V}}\right)^2+\frac{(2-\gamma)}{24}\left(\frac{V}{\bar{V}}\right)^4+\ \ldots\ldots\right],$$

since $\bar{V}^2=\frac{\gamma p}{\rho}$, where ρ is the density when the pressure is p. Hence $(p_0-p)=\frac{k}{2}\rho V^2$, where k may be regarded as the correction factor due to compressibility. It can be readily seen from the above expression that the rate of increase of k with $(V/\bar{V})$ is almost proportional to $(V/\bar{V})$. The values of k when $(V/\bar{V})=1$ and 0·5 are 1·28 and 1·065 respectively.

The above discussion illustrates in a very general manner what happens at the tip of an airscrew, but instead of the air being set in rapid motion, the tip of the airscrew is moving at very high speed into more or less stationary air. Some experiments on an airscrew rotating with a high tip speed at a stationary point have been made by Lynam* at the Royal Aircraft Establishment. He found that as the speed of the blade tip increases towards the velocity of sound the airflow around the airscrew gradually undergoes a complete change, both the inflowing and outflowing streams—more especially the latter—becoming decidedly unstable. When the tip speed is equal to the velocity of sound considerable turbulence is noticed in both streams, the disturbed region being considerably larger than that of the airscrew itself. The outflowing stream entirely disappears;

* "Preliminary report on experiments with a high tip-speed airscrew at zero advance," by E. J. Lynam, A.R.C.Sc. Advis. Comm. Aeron., 1919.

there is in fact evidence of an inflowing stream in the region behind the airscrew disc. The airscrew appears to behave as a centrifugal fan, air being drawn in from both the back and the front and exhausted at the periphery. It should be observed that even although it is only the tip which is moving with the velocity of sound and that the greater part of the blade is working at much lower speeds, it is the flow over the whole disc which is completely changed. No measurements of the thrust were made, but it is to be expected that the almost complete disappearance of the outflowing stream would be accompanied by a decided decrease of thrust. It is rather interesting to note that above a tip speed of 750 ft. per sec., it was found that the B.H.P. of the airscrew, the rotational speed n in R.P.M., and the ratio of the tip speed, V_S, to the velocity of sound $\bar{V}$, were connected by the linear equation,

$$10^{10}\left[\frac{\text{B.H.P.}}{n^3}\right] = -94 + 365\left[\frac{V_S}{\bar{V}}\right].$$

CHAPTER V

SOME METHODS OF MEASURING THE PERFORMANCE OF AN AIRSCREW

INTRODUCTION

MANY experimental researches with model and full-scale airscrews have been made both in this country and abroad. The principal aerodynamical laboratories which have special equipment for testing model airscrews are :—

(*a*) The Aerodynamics Department of the National Physical Laboratory, Teddington.

(*b*) The Royal Aircraft Establishment, Farnborough.

(*c*) The Aerodynamic Laboratory, Stanford University, Cal., U.S.A.

(*d*) The Aerodynamic Laboratory at Auteuil, France.

(*e*) The Aerodynamic Institute at Kouchino, Russia.

(*f*) The Aerodynamic Laboratory of the Brigata Specialisti, Italy.

(*g*) The Aerodynamic Laboratory at Göttingen, Germany.

In each of these laboratories the experiments are made with model airscrews mounted in a wind tunnel. At the National Physical Laboratory, in addition to the wind tunnel equipment, experiments may also be made with airscrews mounted on a dynamometer at the end of a whirling table.

There are three principal methods of measuring the performance of full-scale airscrews :—

(*a*) With a dynamometer mounted on the end of a whirling table. This method is used at the Royal Aircraft Establishment, Farnborough. The first whirling table in this country was erected at Barrow by Messrs. Vickers, Ltd.

(*b*) With the airscrew on a dynamometer mounted on a carriage running on a horizontal track. Experiments with such equipment have been made at the Laboratory of Military Aeronautics at Chalais-Mendon, and also at Saint-Cyr, France, and at Frankfurt, Germany.

(*c*) With the airscrew mounted on an aeroplane in flight. In this case the thrust may be measured directly, either with a thrustmeter or by the integrated difference of total head between the two faces of the airscrew disc, or calculated indirectly from the performance and aerodynamic data of the aeroplane. Usually the torque is calculated from the characteristics of the engine. Such experiments have been made at the Royal Aircraft Establishment and also at the Airship Station at Pulham, Norfolk.

In the present chapter some representative methods of testing both model and full-scale airscrews are described.

METHODS OF MEASURING THE PERFORMANCE OF MODEL AIRSCREWS AT THE NATIONAL PHYSICAL LABORATORY

At the present time there are two different methods of testing model airscrews at the National Physical Laboratory, (*a*) with an airscrew dynamometer mounted on a whirling table and (*b*) with an airscrew balance designed for use in a wind channel. It is proposed to describe both these methods of measuring the performance of an airscrew.

(*a*) *Description of the Whirling Table and Airscrew Dynamometer.*

A brief description of the Whirling Table as first designed is given by Stanton* in the Technical Report of the Advisory Committee for Aeronautics, year 1909–10. Since that time, however, the apparatus has been modified and improved.

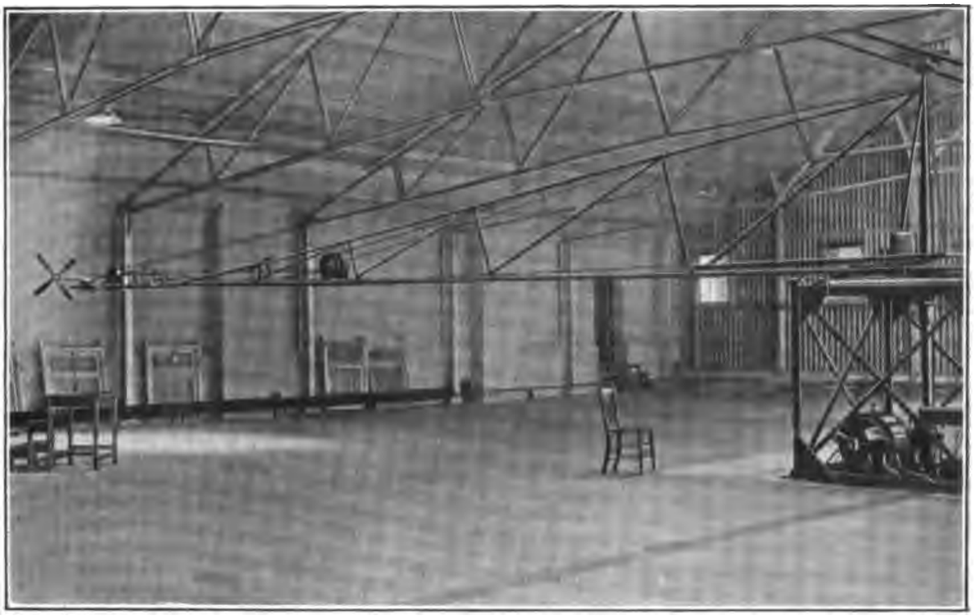


FIG. 17 —Photograph of the Whirling Table at the National Physical Laboratory.

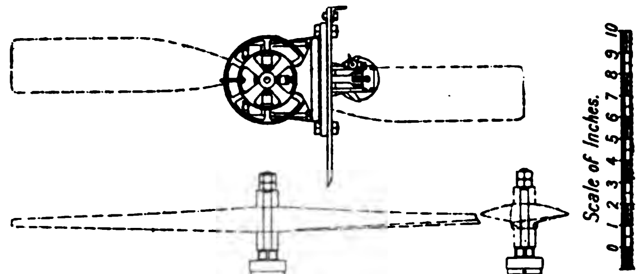
The whirling table, a photograph of which is given in Fig. 17, is housed in a galvanised iron shed 80 ft. square so that the airscrew experiments may be carried on independently of atmospheric conditions. The overall diameter of the whirling table is 60 ft., the airscrew dynamometer being carried at the end of an arm of length 30 ft., which is built up of light steel tubes tapering from 1½ in. diameter at the axis to 1 in. diameter at the extremity. These tubes are spaced 12½ in. apart and connected together by struts. The central post of the whirling table is carried up from the floor to the roof, the arm being stiffened by connecting the tubes to a cantilever built up of light angle iron and

* "Report on the experimental equipment of the Aeronautical Department of the National Physical Laboratory," by T. E. Stanton, D.SC., M.INST.C.E. Advis. Comm. Aeron., 1910.

provided with cross-bars and steel wire ties. The whirling table is driven by a 14 h.p. electric motor, through a worm reduction gear of 28 to 1. To avoid straining the central post by the inertia of the arm when the motor is stopped, the post is cut immediately above the worm wheel and the upper and lower parts connected by a rachet gear which allows the free rotation of the arm. The speed of rotation of the whirling table, which is regulated by an adjustable resistance in the armature circuit of the motor, can be varied from 3 to 17 r.p.m., corresponding with translational speeds at the end of the arm of 6 to 35 m.p.h. For the purpose of transmitting current to the motor driving the airscrew, and also to the measuring apparatus, a set of eight slip-rings are fixed to the central post. The airscrew shaft is driven through an endless rubber belt by a 2 h.p. motor carried on the arm, the speed of the motor being regulated from an observation table in the corner of the whirling shed. A small revolution counter driven by the airscrew shaft is connected electrically so as to ring a bell every hundred revolutions, the airscrew speed being timed with a stop-watch for a period of about two minutes. A second bell rings at each revolution of the whirling table, so that by timing the rings with a stop-watch, the speed of translation of the airscrew due to motion of the whirling table may be calculated. As would be expected, the air in the whirling shed is set in motion by the rotation of the whirling table and also by the backwash of the airscrew, the velocity of translation of the airscrew through the air being obtained from the measurements of both the velocity of translation of the airscrew due to the rotation of the whirling table and the velocity of swirl of the air. This latter velocity is measured by a Pitot tube mounted at the end of a light arm opposite to that carrying the airscrew, the tube following the airscrew at a distance of about 100 ft., so that it is well out of the region of the direct action of the outflowing stream from the airscrew. The Pitot tube is connected through an airtight mercury seal at the top of the central post to one limb of a manometer, the other limb being open to the atmosphere. If V_A be the velocity of the end of the whirling arm and V_S the velocity of the air swirl in the shed, the velocity of the tube relative to the air is $(V_A - V_S)$, so that taking into consideration the reduction of air pressure due to the centrifugal head in the connecting pipe, the reading of the manometer is proportional to $[(V_A - V_S)^2 - V_S^2]$, that is $(V_S^2 - 2V_SV_A)$, from which the velocity of the swirl may be calculated, since V_A is a measured quantity. The speed of the air swirl, when the airscrew is not working, is about 8 per cent of the arm speed. The air swirl depends on both the rotational speed of the whirling table and the outflowing stream of the airscrew. At a low translational speed when the airscrew is giving a big thrust the speed of the air swirl may be negative.

A description of the first airscrew dynamometer is given in the paper previously mentioned. After about twelve months a second dynamometer was designed. The latter dynamometer, which was constructed on the same principle as the former, embodied such improvements in mechanical detail as were suggested from experience, and also permitted measurements to be made over greater ranges of speed, thrust, and torque. A description of this second dynamometer, which is now in use, is given in the Technical Report of the Advisory Committee

for Aeronautics, 1911.* The dynamometer was designed to measure a maximum thrust of 15 lb. and maximum torque of 4 lb.-ft. With model airscrews of scale 1/4th, measurements greater than 4 lb. for the thrust and 1 lb.-ft. for the torque are, however, rarely made.



Two sketches of the airscrew dynamometer—a sectional drawing and a general arrangement—are given in Figs. 18 and 19. From Fig. 18 it will be seen that the airscrew shaft is driven through flat coiled springs, S, the extension of the springs, which is proportional to the torque, being measured by the relative displacement of a pointer over a small drum, a, which carries a small slip of paper. The position of the pointer at any time during the experiment is recorded by passing a series of sparks through the paper so that the magnitude of the torque may be directly calculated from the known calibration of the springs. Alternate torque springs are wound in opposite directions so that tendencies to unwind due to centrifugal force almost counterbalance each other. A small torque correction, the magnitude of which is a function of the rotational speed, is made to each torque measurement. The airscrew shaft is allowed a small axial movement of about 1/200 in. by the use of the ball bearings ppp, which have parallel outer races. The end of the airscrew shaft bears against an oscillating lever, d, which is pivoted at its centre, the lower end of the lever being directly controlled by the thrust spring b, the tension of which is

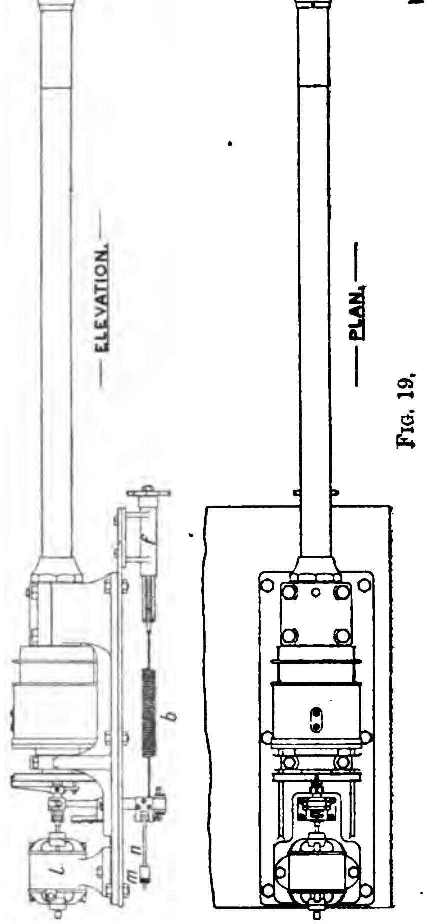
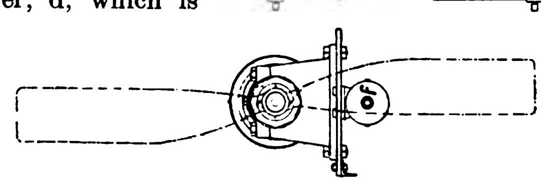


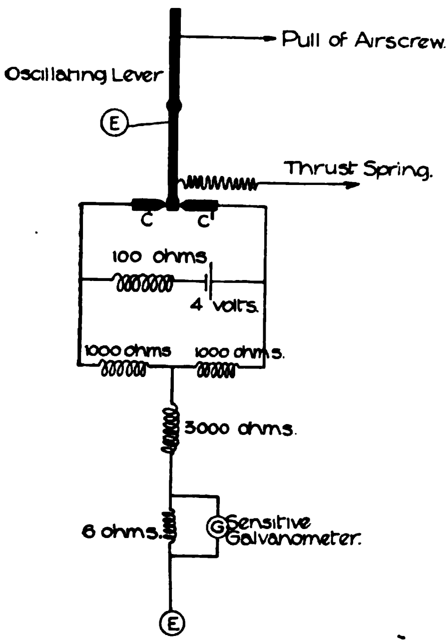
Fig. 19.



* "Experiments on the thrust and efficiency of model propellers, with a note as to a comparison with tests of a full-sized propeller," by L. Bairstow, A.R.C.SC., F. H. Bramwell, B.SC., and W. E. G. Sillick, A.R.C.SC. Advis. Comm. Aeron., March, 1911.

adjusted by the poppet head f and two adjustable stops, c, c′, both stops being insulated electrically from the rest of the dynamometer.

The method of indicating when the thrust of the airscrew is balanced by the pull of the spring is illustrated by the diagrammatic sketch of Fig. 20. From this diagram it is seen that the direction of the current through the galvanometer depends on whether the lever is touching stop c or stop c′. When the thrust of the airscrew balances the pull of the thrust spring and the oscillating lever is floating midway between the two stops, the galvanometer needle is not deflected owing to the inertia of the moving parts and the inductance of the circuit. The sensitivity of this method of balancing is very great, and it will be noted that the accuracy is not affected by any variation of the battery voltage. Moreover, any variation of the contact resistance at either stop is very small compared with the large resistance of the circuit. Each stop has a hardened steel point which makes contact with a small flat hardened steel disc at the bottom of the oscillating lever.



Diagrammatic Sketch of Thrust-Balancing Apparatus.

FIG. 20.

Oscillations in the torque springs due to any small irregularity of the driving torque are damped by the oil dash-pot K. The dash-pot consists of a series of concentric discs, alternate discs being attached to a sleeve carrying the inner ends of the torque springs, and to the drum, to the inside of which are fixed the outer ends of the springs. The vibrations of one end of a torque spring relative to the other are damped out by the viscosity of the oil between the two series of discs.

The plane of the airscrew passes through the axis of rotation of the whirling table, and the centrifugal force of the airscrew has no component in the direction of the thrust. The centrifugal force acting on the airscrew shaft has, however, a small component in the direction of the thrust which is automatically corrected for by the centrifugal force of a small mass m acting through a bell crank lever n.

It is thought desirable to conclude with a few general comments on the accuracy and suitability of this method of testing model airscrews. An experimental investigation* into the accuracy of the airscrew dynamometer showed that from this standpoint the measurement of the airscrew performance was

* "An experimental investigation into the accuracy of the airscrew dynamometer at the National Physical Laboratory," by A. Fage, A.R.C.SC. Advis. Comm. Aeron., December, 1915.

quite satisfactory. The dynamometer had then been in use three years, so that it would appear that any wearing of the rubbing parts was negligible. There is, however, a slight stickiness in the measurement of a small thrust, say up to 0·2 lb. Also there may be some uncertainty in the measurement of a *small* torque because of the difficulty of estimating the " centre of mass " of the spark holes—the length of paper punctuated by the sparks may be equal to the distance between the zero and the centre of the oscillation—and also because the torque correction may be an appreciable percentage of the torque as measured from the diagram. With experience, however, such errors, which nearly always occur at a value of (V/n) without the working range of practice, may be more or less eliminated. The fundamental disadvantage of the airscrew dynamometer is the time and labour needed to make a complete series of airscrew experiments. There are several reasons why this is so. The apparatus, owing to the complications of the working parts, needs constant attention to keep it in good working order. Also the taking of the experimental data is somewhat slow, and when obtained some laborious calculations are needed if the performance of an airscrew is to be presented in a concise manner. Whilst there is no doubt that experiments with airscrews can be made more speedily and conveniently with an airscrew balance in a wind tunnel, it should at the same time be remembered there are some special experiments with airscrews which can only be adequately performed with the aid of a whirling table.

(*b*) *Description of a Balance designed to Measure the Performance of an Airscrew in a Wind Tunnel*

A balance for use with a wind tunnel was designed by Bramwell* for the special purpose of measuring the performance of an airscrew in a lateral wind. As was hoped at that time, the experience gained from the use of this balance —and also from subsequent modifications—has been of assistance in the design of a balance of more general utility.

A photograph of the balance is shown in Fig. 21. The principle of the method of measurement is to " weigh " directly both the thrust and the torque. A sketch is given in Fig. 22 showing the general arrangement of the balance when supported on the roof of a wind tunnel. It will be noticed that the balance arm, A, which projects downward and is of such a length that the airscrew is at the centre of the tunnel, carries at its lower end the bearings, B, for the horizontal airscrew shaft, C. The weighings are made on two horizontal arms at right angles, E, the axes of which pass through a single conical point, D, which supports the weight of the balance. The motor F was mounted on the balance above the point of support, and partly counterbalanced the weight of the balance arm, airscrew, etc.; additional weights to adjust the stability of the balance could be added on a short vertical spindle, G, above the motor. The connections from the regulating resistances to the motor were made through small mercury cups. The drive

* " Experiments to determine the lateral force on a propeller in a side wind," by F. H. Bramwell, B.SC., E. F. Relf, A.R.C.SC., and L. W. Bryant, A.R.C.SC. Advis. Comm. Aeron., March, 1914.

from the motor to the airscrew shaft was by means of a round leather belt passing through a slot in the roof of the wind tunnel. The inclination of the airscrew shaft to the wind direction could be varied by rotating the whole of the balance on the base plate M. One measuring arm was thus always parallel and the other always at right angles to the axis of the airscrew shaft.

With the airscrew shaft parallel to the wind direction, the thrust was measured on the arm parallel to the airscrew axis and the torque on the arm at right angles to the airscrew axis. With the balance rotated so that the airscrew shaft was inclined to the wind direction, the thrust was measured on the arm parallel to the airscrew axis, but the reading on the other arm was due partly to the torque and partly to the lateral force on the airscrew. When measuring a lateral force, readings were taken with the airscrew shaft inclined at equal angles to the symmetrical position, that is the wind direction. The mean of the readings on the arm at right angles to the airscrew axis was then a measure of the torque of the airscrew at this angle of yaw, and half the difference was a measure of the lateral force. The airscrew was mounted close to the vertical axis of the balance, so that after a rotation about this axis through a considerable angle the airscrew was still appreciably in the centre of the tunnel. The balance was prevented from rotating during an experiment by means of a strut and "C" spring. To damp out oscillations an oil dash-pot was placed between the point of support of the balance and the roof of the wind tunnel. The dash-pot also served the purpose of an oil seal to prevent a flow of air into the tunnel. To measure the rotational speed a small toothed wheel engaged with a worm on the shaft, so that an electrical contact was made every 54 revolutions of the airscrew; this contact was in series through one of the mercury cups with a bell and battery, the rings of the bell being timed with a stop-watch. With the balance as first designed it was necessary to measure the corrections for the wind forces on the balance arm and the support for the airscrew shaft. These corrections were determined by removing the bearings from the airscrew shaft and mounting the shaft independently from the floor of the channel by two pivots supported on light struts. With this method of

FIG. 21.—Photograph of Airscrew Balance.

mounting, the airscrew shaft passed freely through the bearing housings so that the balance was quite free. The necessary corrections were then measured at conditions the same as those of the experiment.

(c) *Modification of this Airscrew Balance*

As stated previously, the present balance was designed for the special purpose of measuring the lateral force on an airscrew. It was necessary then for the airscrew to be mounted quite close to the vertical axis of rotation of the balance. The balance was, however, unsatisfactory for the measurement of the ordinary performance of an airscrew, with or without the interference of a body. To eliminate several causes of inaccuracy of measurement, the balance was subsequently considerably modified. The first modification was the elimination of the rather large corrections—due largely to the wind forces acting on the vertical arm of the balance and the bracket carrying the airscrew shaft, and also to the interference between the vertical arm and the airscrew—which had to be applied to the readings of both the thrust and the torque. Accordingly the airscrew shaft was lengthened so that the airscrew was about 15 in. forward of the vertical arm of the balance. The part of the balance within the wind tunnel was then shielded from the wind by enclosing the vertical arm in a brass tube, and the casting carrying the shaft bearings in a small wooden streamline body. Both the guard tube and the streamline body, which were rigidly attached to the wind tunnel, were of course detached from the moving parts of the balance.

A modification of the method of measuring the torque was also made. With the balance as first designed, the torque was measured about an axis parallel to but 36 in. above the airscrew axis. It follows, then, that any small lateral wind force on the lower end of the balance due to a slight error in the setting of the airscrew axis along the wind direction, or a small component of the thrust at right angles to the axis about which the torque is measured, due to an error in the alignment in parallel of this axis and the airscrew axis, are measured with the torque ; and since these small forces have the long leverage of 36 in. they may seriously affect the accuracy of the torque measurement. The obvious method of eliminating this inaccuracy was to measure the torque about an axis as close as possible to and parallel with the airscrew axis. Accordingly, then, the airscrew balance was supported on two hard steel points bearing in cups at the ends of the casting carrying the airscrew shaft. The motion of the balance was thereby constrained to be one of slight oscillation about an axis parallel to and about 1·25 in. below the airscrew axis. With this method of support the C.G. of the balance is above the axis of rotation, so to make the oscillations stable, springs were introduced connecting the top of the balance with the tunnel.

It may be well to outline briefly this method of measuring the performance of an airscrew. As previously stated, when measuring the thrust the balance is mounted on the single top point, the motion in a plane perpendicular to the air-

screw axis being prevented by a small strut placed between the torque arm of the balance and the supporting framework. A weight is now hung on the thrust arm, the rotational speed of the airscrew being adjusted until the thrust of the airscrew balances the load on the arm. Measurements are then made of both the rotational speed and the wind speed in the channel. To measure the torque the balance is lifted off the top point by screwing up the two points below the airscrew axis. The stability springs are mounted in place at the top of the balance and the measurement of torque is made in a manner similar to that of the thrust, that is by adjusting the rotational speed of the airscrew until the torque balances the moment of the weight at the end of the " torque " arm.

In conclusion it should be stated that a new airscrew balance embodying improvements suggested by experience is now being constructed at the National Physical Laboratory.

(d) *Description of an Apparatus for the Measurement of the Performance of an Airscrew in a Wind Tunnel*

It is now proposed to describe an apparatus* used to measure, in conjunction with the ordinary balance of a wind tunnel, the performance of a model airscrew when mounted in position on a model of an aeroplane body. The apparatus is equally suitable for the measurement of either the windage torque of a model of a rotary engine or the performance of a combination of a model airscrew and a model of a rotary engine.

A sketch of the apparatus as arranged to measure the windage forces on a model of a rotary engine when rotating behind and with the same speed as the model airscrew is given in Fig. 23. The airscrew would, of course, occupy the position of the rotary engine if its performance were being measured. The apparatus consists of a 1½ h.p. electric motor (A) bolted to a cradle, to the ends of which are attached two hardened steel points bearing in the cups of the " Y " pieces (M). These " Y " pieces are rigidly connected to the lower ends of the diagonally arranged wires (C), the upper ends of which are supported from the roof of the channel by stirrups carrying hardened steel points bearing in the cups (D). With this method of support the electric motor with either the airscrew or the rotary engine as the case may be would be capable, if there were no other constraint, of a swinging motion in the direction of the motor shaft, and also of a rocking motion about the axis passing through the points BB, this axis being parallel to and slightly below the axis of the shaft. A spindle arm, E, projecting from the under side of the cradle is connected by a strut and " C " spring to the top of a spindle clamped in the top of the trumpet of the channel balance. The spindle arm E and the top of the balance trumpet are enclosed in a guard which is cylindrical over the greater part of its length, but of a streamline shape where surrounding the strut and " C " spring. To avoid any unneces-

* " Description of apparatus for measurement in a wind tunnel of the performance of an airscrew or the windage torque of a rotary engine," by A. Fage, A.R.C.SC., H. E. Collins, and T. H. Fewster. Advis. Comm. Aeron., 1918.

sary constraint of the moving parts of the measuring apparatus, electrical connection between the motor and the mains is made through the standard form of mercury cups (G). A revolution counter (H), driven by the motor shaft and in electrical connection with a bell, enables the rotational speed of the airscrew to be measured. The scale of the models to be used with the apparatus needs to be such that the electric motor with its contiguous parts are completely enclosed in a shell of the model aeroplane body (K), and such as to ensure a sufficient clearance between the surrounding shell and the moving parts of the measuring apparatus. The body surrounding the motor is supported by two iron bars (L) secured at their ends to the walls of the channel.

Measurement of Torque.—When measuring torque, the rocking axis BB of the motor and airscrew, which is parallel to the airscrew shaft, is fixed parallel to the centre line of the channel by rigidly attaching the " Y " pieces to the cross-channel bars (L).

The brackets at the lower end of the arm (E) and at the top of the balance spindle are adjusted so that the strut which transmits the load to the top of the balance is at right angles to the axis of the airscrew shaft. A direct calibration of the apparatus may be made by applying a known torque and weighing directly with the balance.

The distance between the rocking axis BB and the axis of the airscrew and motor is small—about 1 in.—so that a component of the thrust at right angles to the rocking axis due to any want of parallelism between these two axes will affect only slightly the measurement of torque.

Measurement of Thrust.—To measure the thrust the " Y " pieces are detached from the fixed bars (L) so that the motor and airscrew have freedom to swing in a longitudinal direction about the points (D) at the upper ends of the wires and the points (B) of the motor cradle. The brackets on the lower end of the arm (E) and the top of the channel spindle are adjusted so that the strut (F) which transmits the thrust load to the top of the balance is parallel to the airscrew axis. A strut (N) which is held in position with a " C " spring, between the bracket (O) of the motor cradle and one of the bars (L), prevents any rocking of the motor about the axis BB. This strut is mounted at right angles to the direction of the thrust so that no force component of the torque enters into the measurement of the thrust and no constraint to small longitudinal oscillations of the motor is introduced. The calibration of the thrust-measuring apparatus may be obtained by applying a known thrust along the airscrew axes and measuring directly on the airscrew balance.

Measurement of the Forward Velocity of the Airscrew.—It will be apparent when the area of the airscrew disc is large compared with the cross-sectional area of the tunnel, that to obtain the velocity of the airscrew relative to undisturbed air, a correction has to be applied to the mean wind velocity in the tunnel to allow for the air drawn into the tunnel by the working of the airscrew.

(e) *Apparatus for Rotating an Airscrew on an Independent Shaft.*

It is often necessary, such as when measuring the extra resistance of an aeroplane body due to the working of the airscrew, to mount the airscrew on an independent shaft. A sketch of an apparatus designed for such a purpose is given in Fig. 1. The airscrew shaft, which runs in two gun-metal bearings rigidly attached to the channel with bars and tie wires, is driven through leather belting from a motor mounted on the top of the channel. The speed of the airscrew is measured with the aid of a worm wheel and a worm on the airscrew shaft, which is in electrical connection with a bell.

The Method of Testing Model Airscrews at the Aerodynamic Laboratory at the Leland Stanford Junior University

A complete description of the special equipment* installed at the Aerodynamic Laboratory at the Leland Stanford Junior University for the testing of model airscrews is published by the National Advisory Committee for Aeronautics, U.S.A. The design of the Aerodynamic Laboratory was the joint work of Dr. W. F. Durand and Prof. E. P. Lesley. A sketch showing the general arrangements of the apparatus in a wind tunnel of the Eiffel type is given in Fig. 24. The special equipment for airscrew testing consists of (*a*) a thrust dynamometer, (*b*) a torque dynamometer, (*c*) a revolution counter, and (*d*) an air-speed meter.

(*a*) *The Thrust Dynamometer*

A general arrangement of this apparatus is shown in the drawing of Fig. 25. The apparatus was so designed as to place the airscrew approximately 1½ diameters forward of any appreciable obstruction to the outflowing stream. The airscrew is carried on the forward end of a shaft of diameter 1¼ in., which runs in ring-oiling cylindrical bronze bearings. The shaft is driven without any longitudinal constraint through a yoke at the rear end, having hardened steel flat longitudinal surfaces which engage with small ball-bearing rollers on a companion yoke carried by a bevel gear. This bevel gear runs on ball bearings outside the hub, which is bored to provide freedom from contact with the airscrew shaft. The driving motor of 7·5 h.p. is placed at the side, well out of the wind stream, and drives the airscrew shaft through bevel gearing and the yokes mentioned. In this manner the airscrew shaft whilst rotating is free to move longitudinally under the action of the thrust and the controlling or balancing longitudinal forces. To measure the thrust the airscrew shaft is furnished with two thrust ball-bearings which connect through hardened steel knife edges with a vertical lever, which is attached to a horizontal shaft. This latter shaft carries, well beyond the wind stream, a horizontal scale beam with appropriate weights. By adjusting the position of a weight at the bottom of the vertical lever the sensitivity of the measurement of thrust may be varied. The range of travel of the vertical arm,

* "The Aerodynamic Laboratory at Leland Stanford Junior University and the equipment installed, with special reference to tests on air propellers," by W. F. Durand. National Advisory Committee for Aeronautics.

and hence the longitudinal movement of the airscrew, is controlled by suitable stops.

(b) *The Torque Dynamometer*

The general arrangement of the torque dynamometer is shown in Fig. 26. The motor shaft is extended to the casing of the thrust dynamometer stand and

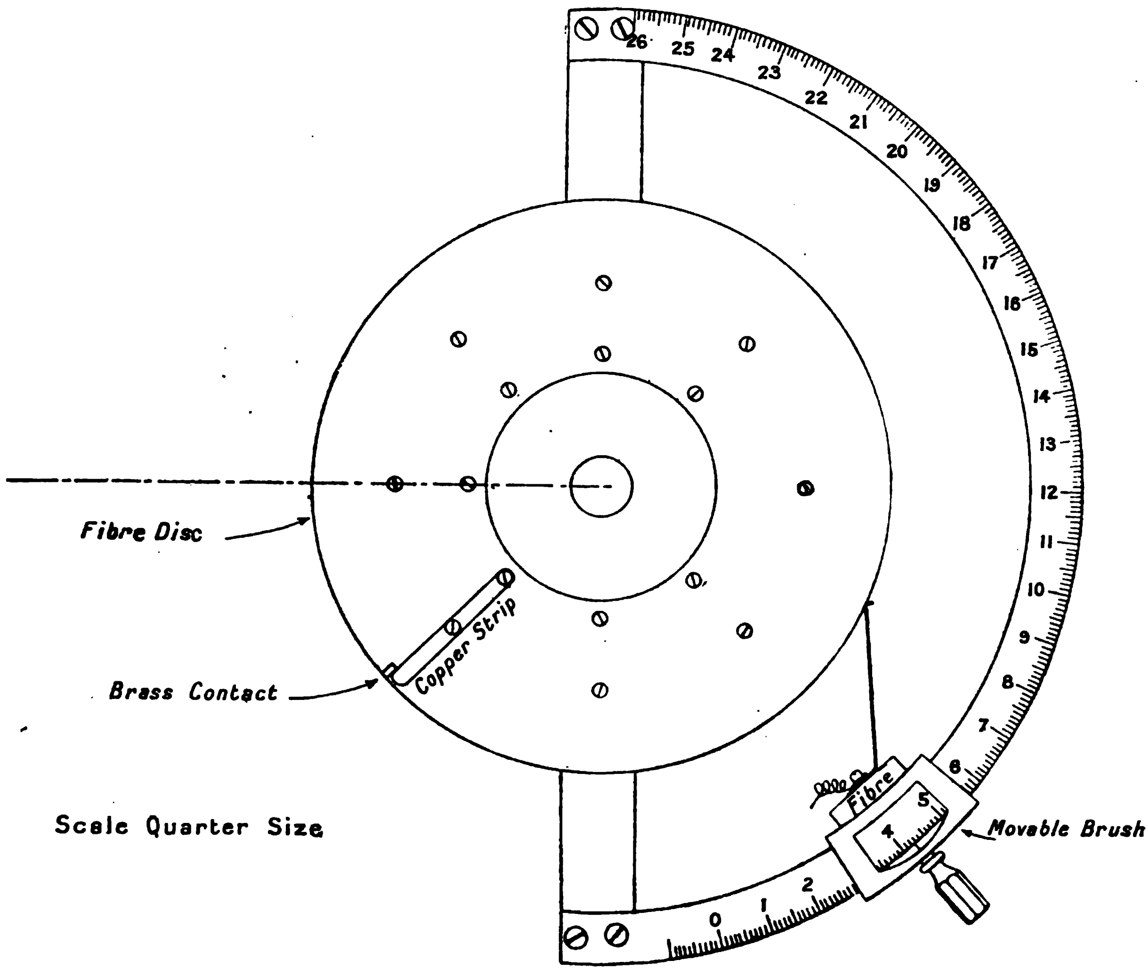


FIG. 26A.

is cut for the insertion of a special coiled spring, which measures the torque on the motor shaft. To measure the torsion of the coiled spring two fibre discs are fitted to the shaft, one on each side of the spring, actually at A and B, as shown in the figure. These discs carry a narrow metal strip on the edge to serve as an electric contact. The contacts are electrically connected to the shaft, and hence to each other. A fixed brush resting on the face of the disc A is carried by the dynamometer frame. This brush is connected through a battery, telephone

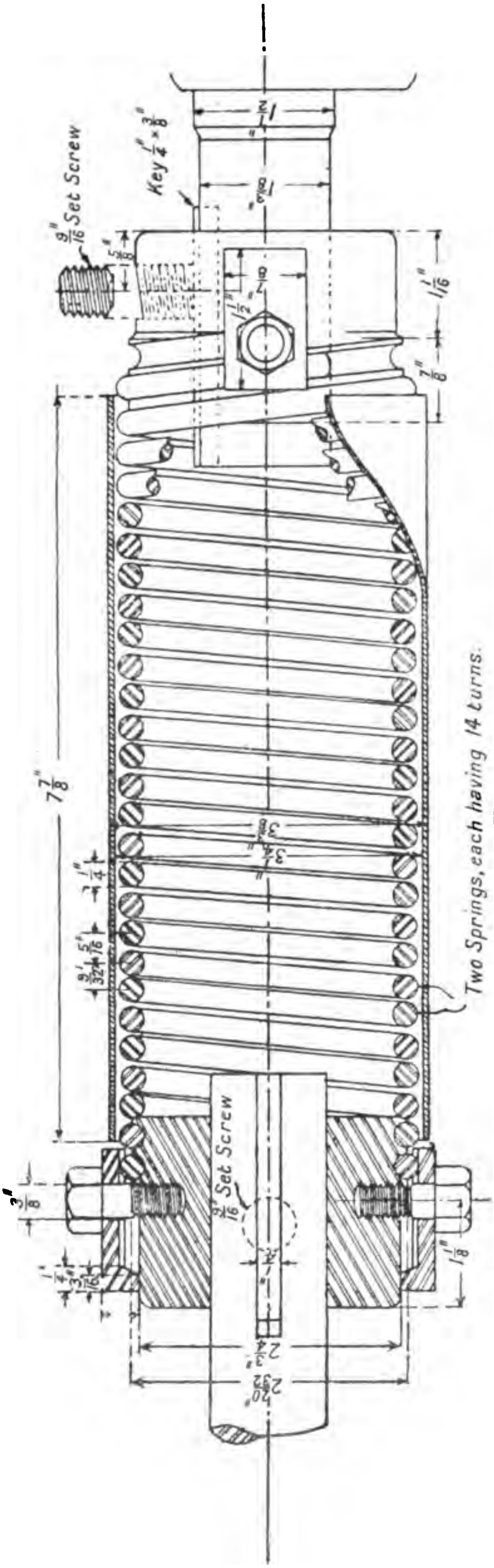


FIG. 26B.

receiver to a second brush mounted over the disc B. When the contacts on the discs pass simultaneously under the brushes the electric current is closed for an instant and a click is heard in the telephone receiver. If, then, with no torque on the shaft the brush carrier at B is adjusted to give simultaneous contacts, a click in the receiver is heard; with a torque the spring twists so that in order that a click may be again heard it is necessary to move the brush holder at B to a point where the contacts will again be simultaneous. The torsion dynamometer was calibrated with a Prony Brake attached to the shaft at the airscrew position. The loads which were applied at various speeds were varied from zero to the full capacity of the driving motor.

(c) *The Revolution Counter*

The revolutions are counted by the movement on a drum geared down by a double worm-gear drive and so adjusted in diameter that 1 in. of travel on the face of a paper strip carried on the drum is just 50 revolutions. The drum is appropriately mounted on a frame with pencil carrier and with electric connection to a seconds pendulum. In operation, the drum revolves and the pencil resting on the paper draws a line with jogs introduced by the click at the second intervals. From the diagram thus obtained it is, therefore, a simple matter to calculate the rotational speed.

(d) *The Air-speed Meter*

The ultimate measure of the air speed was made with the Pitot tube. During the experiments, however,

it was not convenient to make the observations of velocity with the Pitot tube, and accordingly measurements were made between the difference of pressure outside and within the experiment room. It was found that there was a definite relation between this difference of pressure and the air velocity in the tunnel. The difference of pressure was weighed with a sensitive balance which consists of two manometric cells carried on opposite ends of a lever pivoted at its centre. The lower ends of the cells dip under coal oil, the space above the liquid being connected in one case to the air in the experimental chamber and in the other to the air in the room. The zero position of the scale is determined with both cups connected to the outer room. Coal oil is used because it keeps the cans wet and there is no variation in the meniscus.

Description of the Whirling Table for Testing Full-Scale Airscrews at the Royal Aircraft Establishment, Farnborough*

A photograph of this whirling table is shown in Fig. 27, from which it will be seen that the arm is a steel cantilever about 150 ft. long, pivoted at one end and supported near its centre of gravity on two trucks running on a mono-rail track of 50 ft. radius. The centrifugal force on the arm, which is about 30 tons at a tip speed of 80 m.p.h., is taken on a roller bearing at the pivot, which is of cast steel bolted down to a concrete base. The arm is driven by four electric tram motors, each of 50 h.p., which drive through gearing, the four wheels of the two trucks running on the circular mono-rail track. With these four motors working at full power, and also with the airscrew working, a speed at the end of the arm of 80 m.p.h. can easily be attained. To minimise both the power consumed and the interference of the arm on the performance of an airscrew, the outer parts of the whirling arm are suitably faired.

The airscrew is mounted at the end of the arm about 20 ft. above the ground level. The power needed to drive the airscrew is transmitted through a radial shaft 150 ft. long, a bevel gear box, and a dynamometer from a 150 h.p. electric motor situated on the arm near the pivot. The range of the rotational speed of the airscrew is from 600 r.p.m. to 1400 r.p.m.

The underlying principle of the thrust and torque dynamometers is to balance both the thrust and the torque by adjustments of the oil pressure in suitably arranged cylinders. With the torque dynamometer, of which a sketch is given in Fig. 28, there are two pairs of opposed cylinders arranged circumferentially on a disc driven from the gear box. One cylinder of each pair transmits the drive, whilst the other is connected to the recording instrument in such a manner as to compensate for the centrifugal force acting on the oil. These cylinders drive a floating frame and in turn a second disc in such a manner that the whole forms a universal joint, which allows the large axial movement needed for the

* Permission to publish this information has been granted by the Director of Research, Air Ministry, and also by the Superintendent of the Royal Aircraft Establishment.

The author also wishes to acknowledge the kindness of Mr. R. McKinnon Wood, of the Royal Aircraft Establishment, who placed at the author's disposal a complete description of the whirling table.

Fig. 27.

working of the thrust dynamometer. This latter disc is connected directly to the airscrew by a shaft passing through a non-rotating tube, the thrust being transmitted from the shaft to the tube through a thrust ball-bearing. As shown diagrammatically in Fig. 28A, the thrust on the tube, which has freedom of movement in the direction of its length, is taken by the pressure of the oil in a cylinder. As with the torque, a second cylinder is mounted to compensate for the pressure due to centrifugal force acting on the oil. The oil pressure in the dynamometer cylinders is maintained by a pump. With the correct oil pressure, the force on a dynamometer piston just balances the external thrust, so that the piston moves forward and in so doing uncovers ports in the side of the cylinder, through which

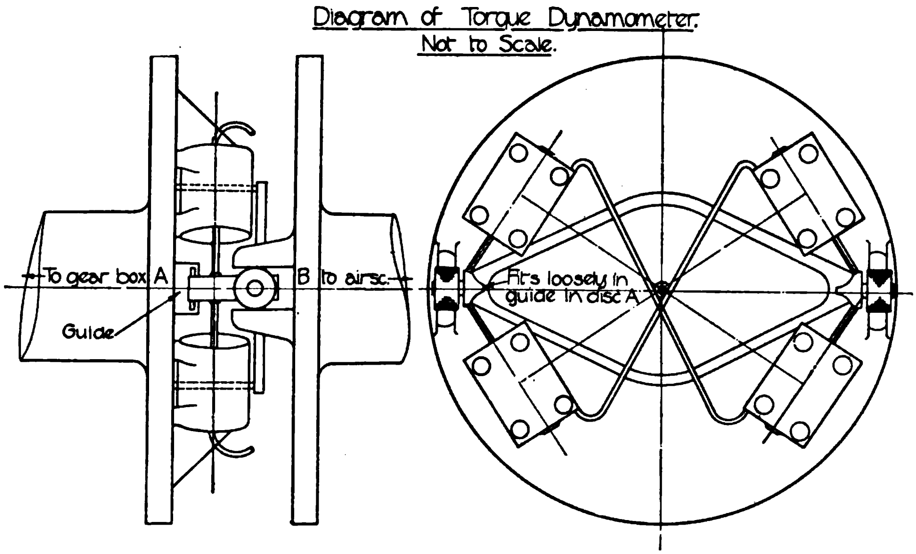


FIG. 28.

the surplus oil escapes. When measuring the thrust the piston floats freely, provided that the pump can supply more oil than that lost by any unpreventable leakage. The oil pressures are measured directly by Crosby Gas Engine Indicators, the correction for the centrifugal head due to the rotation of the arm being automatically made by a suitable connection with the oil pressure of the corresponding idle cylinder. The maximum pressure of the oil is about 450 lb. per square inch. To eliminate any error due to the ordinary pressure gradient of flowing oil, two pipes are fitted to each dynamometer cylinder, one carrying the oil from the pump and the other transmitting the pressure to the Crosby Indicator.

During an experiment automatic records of thrust, torque, time, and revolutions of the arm and of the airscrew shaft are made by small stylograph pens tracing on revolving paper drums.

Ordinary steam-pressure gauges, which are free from any appreciable lag, are fitted to both the thrust and the torque systems. Also the rotational speed of the airscrew can be immediately read from a revolution meter. At the end

of the arm is an ordinary Pitot and Static Pressure tube which is connected to an aeroplane air-speed indicator in the observation cabin. The combination of the Pitot and Static Pressure tubes measures the speed of the end of the arm relative to the air, whilst the Static Pressure tube measures the speed of the end of the arm relative to the ground.

With both the arm and airscrew rotating at the desired speeds, continuous records of thrust, torque, and rotational speeds are taken. On all but the very calmest days these records vary with the angular position of the whirling arm, largely because—in so far as the airscrew is concerned—the direction of the

Diagram of Oil System on Whirling Arm

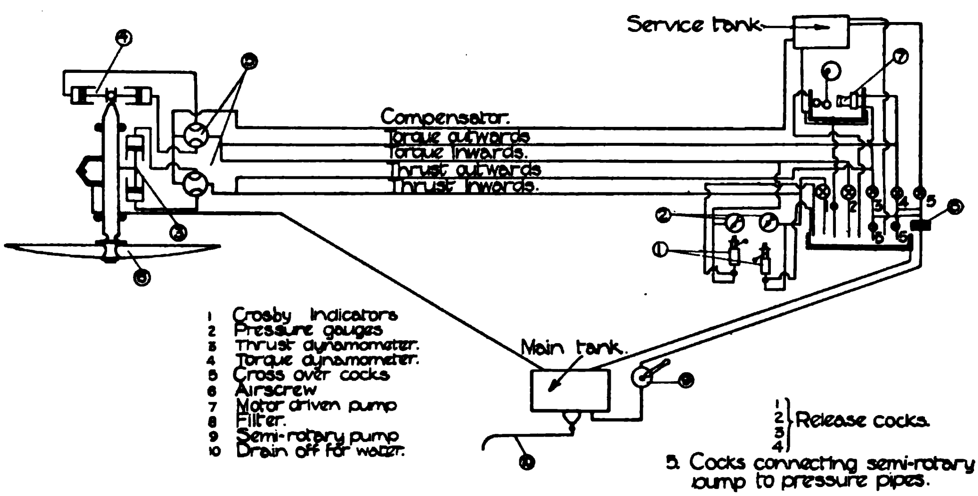


FIG. 28A.

natural wind varies, and also because of the normal fluctuations of the speed of the natural wind. There will also be secondary fluctuations because of the small variations of the resistance of the arm due to changes of the speed relative to the air.

It has been found, however, that the observations are very consistent if the maximum speed of the natural wind is not greater than about 15 m.p.h.

Airscrews rotating below 1400 r.p.m. can be tested at actual climbing conditions, and by using the reduction gear the performance of an airscrew over the complete working range of (V/n) can be measured, although not at the speeds of practice.

CHAPTER VI

ON THE PERFORMANCE OF AN AIRSCREW

Methods of Presenting Performance Data

Having described at some length several methods of measuring the performance of an airscrew, the next step is to discuss the various methods of collecting data as measured directly from such experiments. Generally, the experimental observations are not in a convenient form for the ready calculation of the performance of an airscrew at the working conditions of practice. In fact the large number of parameters involved renders the exact appreciation of the performance a more or less difficult matter. At most of the aerodynamic laboratories the airscrew data are expressed in the compact and convenient system of absolute coefficients. An absolute coefficient has the great merit that its value is independent of the system of units employed so long as the units themselves are dynamically consistent. Absolute coefficients have therefore no dimension and so are very convenient for an international comparison of experimental data. Moreover, the airscrew performance as calculated from such coefficients may be readily expressed in the practical units of either the English or the Metric systems.

With the accepted notation, namely,

V = translational speed of the airscrew,
n = rotational speed of the airscrew,
T = thrust of the airscrew,
Q = torque of the airscrew,
D = diameter of the airscrew,
and ρ = density of the air,

we may write either

$$T = k'_T \rho V^2 D^2 \quad \text{or} \quad T = k_T \rho n^2 D^4,$$

so if T, ρ, V, n, and D are expressed in any system of consistent units k_T and k'_T have no dimension and so are absolute coefficients. In text-books on Dynamics the units of force, mass, length, and time are on the poundal, pound, foot, and second, but the lb.-slug-foot-second system of consistent units is usually employed by the engineer. Accordingly, expressing the density of the air in slugs per cubic foot—$\rho = 0{\cdot}0762$ lb. per cubic foot $= \dfrac{0{\cdot}0762}{32{\cdot}2}$, that is, $0{\cdot}00237$ slugs per cubic foot at a temperature of $15{\cdot}6°$ C. and pressure 760 mm.—the value of the thrust in lb. may be calculated from the value of k_T or k'_T, if V be expressed in ft. per sec., n in revs. per sec., and D in ft. In a similar manner we may write either $Q = k'_Q \rho V^2 D^3$ or $Q = k_Q \rho n^2 D^5$. Since $k_T = k'_T (V/nD)^2$ and $k_Q = k'_Q (V/nD)^2$,

the airscrew performance may be expressed completely by plotting either k_T and k_Q against (V/nD) or k'_T and k'_Q against (V/nD). The efficiency which is the ratio of the useful work done to the total work put into the airscrew is also an absolute coefficient.

$$\text{Thus efficiency } \eta = \frac{\text{Thrust} \times \text{Translational speed}}{2\pi \times \text{Torque} \times \text{Rotational speed}}$$
$$= \frac{k_T}{2\pi k_Q}\left(\frac{V}{nD}\right) \text{ or } \frac{k'_T}{2\pi k'_Q}\left(\frac{V}{nD}\right).$$

We see, then, that the performance of an airscrew may be expressed completely in two systems of absolute coefficients—either $\left(k_T, k_Q, \frac{V}{nD}, \eta\right)$ or $\left(k'_T, k'_Q, \frac{V}{nD}, \eta\right)$. The former system is probably the better because k_Q, that is $(Q/\rho n^2D^5)$, may also be regarded, when the diameter of the airscrew and the performance characteristics of the engine are known, as an absolute coefficient of the engine.

FIG. 29.

To illustrate the usefulness of this system of absolute coefficients, it is proposed to calculate from such coefficients the performance of the airscrew of which a photograph is given in Fig. 29. The diameter of the full-scale airscrew is 9 ft. 2 in. The performance data of Fig. 30 were calculated from the data of experiments made with a model of scale 1/4th.

Suppose we wish to know the values of the thrust and the torque of the airscrew at ground level, when it is moving through the air with a translational speed of 75 m.p.h. and a rotational speed of 1200 r.p.m. Firstly, we express the known data in consistent units, so that

$V = 110$ ft. per sec.

$n = 20$ revs. per sec.

$D = 9{\cdot}167$ ft.

$\rho = 0{\cdot}00237$ slugs per cubic foot (assuming the temperature of the air to be $15{\cdot}6°$ C. and the pressure 760 mm.).

The value of V/nD at which the airscrew works is $\left(\frac{110}{20 \times 9{\cdot}167}\right)$, that is 0·60, the corresponding values of $\left(\frac{T}{\rho V^2 D^2}\right)$ and $\left(\frac{Q}{\rho V^2 D^3}\right)$ as measured from the curves being 0·157 and 0·0209 respectively.

Hence $T = 0{\cdot}00237 \times (9{\cdot}167)^2 \times (110)^2 \times 0{\cdot}157 = 378$ lb.

and $Q = 0{\cdot}00237 \times (9{\cdot}167)^3 \times (110)^2 \times 0{\cdot}0209 = 460$ lb.-ft.

$$\text{The efficiency} = \frac{VT}{2\pi Qn} = \frac{110 \times 378 \times 100}{2\pi \times 460 \times 20} = 72 \text{ per cent.}$$

The thrust and torque could, of course, be calculated from the values of $(T/\rho n^2 D^4)$ and $(Q/\rho n^2 D^5)$.

Discussion of "Pitch" and "Slip"

The geometrical pitch of a blade element is the distance it would advance in one revolution if it were moving in the direction of the chord. (See Fig. 1.) The geometrical pitch is, therefore, equal to $2\pi r \tan \theta$, where r is the distance of the blade element from the axis of rotation and θ is the blade angle. The helical path described by a point on the chord would be similar to the thread of a bolt, so that the geometrical pitch is analogous to that of an ordinary nut and bolt. In airscrews as now designed the geometrical pitch is not a constant for all the blade elements, so that it is necessary to introduce another conception in order that pitch may be characteristic of the airscrew as a whole rather than of any particular section of the blade. Returning to the case of an ordinary nut and bolt, the pitch of the nut as it is screwed on the bolt is the distance which it advances along the axis of the bolt during each revolution, the bolt usually being rigidly fixed. During the relative motion the shape of either solid is not altered by the forces acting. With an airscrew, however, we have the case of a solid working in a highly rarefied medium which readily yields to pressure and to which motion may be readily imparted. Supposing the airscrew to be moving through the air at such forward and rotational speeds that it is giving no thrust, then the forces acting on the air are small, and we may consider the air to be more or less stationary, so that from the analogy of the nut and bolt in which we regarded the bolt as fixed, we may define the mean pitch as the distance moved forward in one revolution when the airscrew is giving no thrust.

This mean pitch which can only be measured experimentally is usually referred to as the experimental mean pitch. Although the thrust is zero, the air will have some slight disturbance due to the forces needed to turn the airscrew, and also because the zero thrust is the sum of the small positive and negative thrusts of the several parts of the blade. The experimental mean pitch is therefore an idea which does not admit of precise physical definition, and its use in the presentation of the data of airscrew experiments is somewhat superfluous.

An airscrew which is giving a thrust cannot be moving through still air, since the thrust is the reaction from the air which is pushed backwards. Also the airscrew must be supported or balanced on air which has a general backward motion

due to the thrust, that is the airscrew slips and so cannot grip or bite rigidly, as is the case with the nut and bolt. It follows, then, that when an airscrew is giving a thrust the forward advance during one revolution is less than the experimental mean pitch, the slip being measured by the difference between these two quantities. The slip-ratio may be defined as the difference between the forward advance during one revolution when the airscrew is giving a thrust and the experimental mean pitch, expressed as a fraction of the experimental mean pitch.

In the writer's opinion the notion of slip is superfluous ; and the introduction of a slip-ratio as a performance parameter is quite unnecessary.

Discussion of the Performance Data of a Representative Airscrew

The curves of Fig. 30 completely define the performance, over the working range of practice, of the airscrew under consideration. To make the present dis-

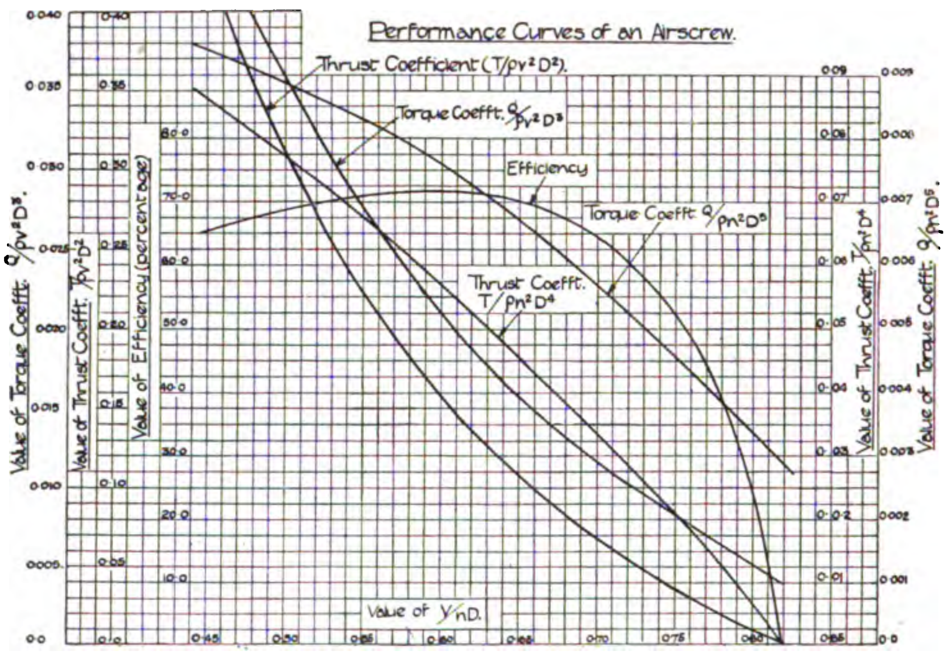


Fig. 30.

cussion complete, however, these curves have been extended over a large range of (V/nD), as shown in Fig. 31. It will there be seen that the thrust coefficient has its maximum value when the airscrew is rotating at a stationary point (V/nD) $=0$, and a zero value when (V/nD) $=0{\cdot}82$, in which case the forward advance per revolution is equal to the experimental mean pitch of 7·5 ft. With an increase from the zero value of (V/nD) the torque coefficient at first increases very slightly and afterwards decreases very rapidly. The hump in the torque curve would probably be more pronounced in the case of an airscrew with larger blade angles. Obviously the airscrew has zero efficiency when (V/nD) $=0$, since $V=0$, and

also when $(V/nD)=0{\cdot}82$, in which case $T=0$. From the diagram it will be seen that at any value of (V/nD) the airscrew has only one value of efficiency. The maximum efficiency is 72·0 per cent, and occurs when $\left(\frac{V}{nD}\right)=0{\cdot}60$. When testing a model of this airscrew we found over the speed range of experiment that at any value of (V/nD), both the thrust and the torque were each proportional to either the square of the rotational speed or the square of the translational speed. Also with the airscrew rotating at a stationary point both the thrust and the torque were each proportional to the square of the rotational speed.

The curves of Fig. 32 show the relationship between the translational and

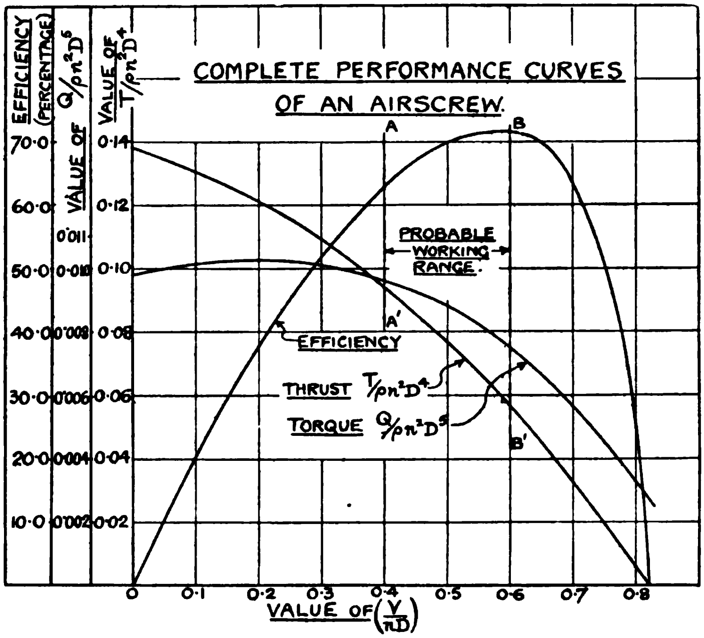


FIG. 31.

rotational speeds of this airscrew at constant values of the thrust. It will be noticed that if the translational speed be increased then the thrust falls unless the rotational speed is also increased. With any line such as OAB passing through the origin the ratio of the forward speed to the translational speed is a constant. Taking any two points—such as A and B—of intersection of this line with the lines of constant thrust, it will be seen that the ratio of the thrusts is equal to the ratio of the squares of either the translational or the rotational speeds. From the curves of Fig. 33 it follows, if the thrust be maintained at a constant value, that an increase of translational speed is also accompanied by an increase of torque.

Relationship between the Translational and Rotational Speeds at a Constant Thrust of an Airscrew.

Diameter of the Airscrew = 9 ft. 2 in.

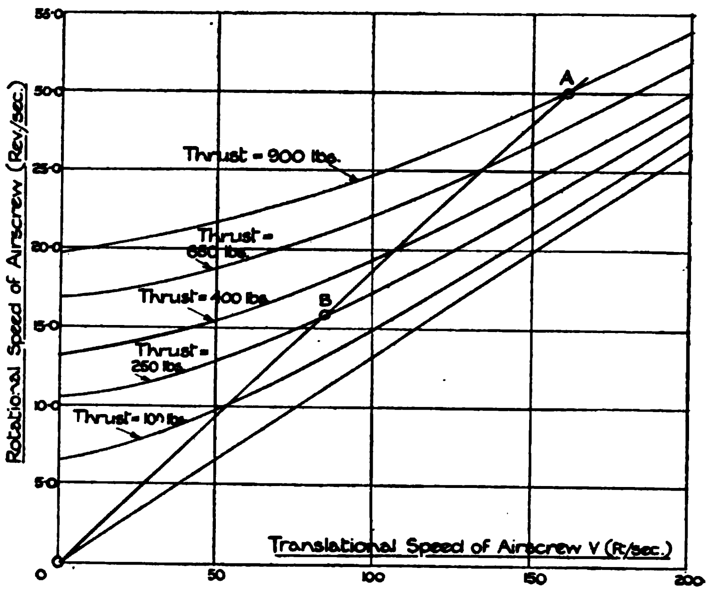


FIG. 32.

Relationship between translational speed and torque at constant thrust of an airscrew.

Diameter of the airscrew ~ 9' 2"

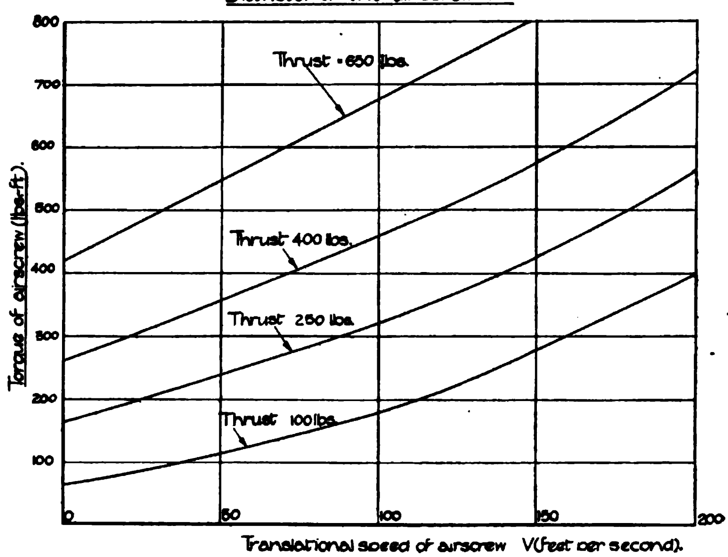


FIG. 33.

General Formulæ for the Thrust and Torque Coefficients of an Airscrew

Bairstow* has found that the performance of an airscrew may be expressed with good accuracy by the following formulæ :—

$$C_T\left(\frac{T}{\rho n^2 D^4}\right)=\frac{4}{3}\left[1-\left(\frac{V}{np_e}\right)^2\right]$$

and $$C_Q\left(\frac{Q}{\rho n^2 D^5}\right)=1\cdot104-0\cdot833\left(\frac{V}{np_e}\right)^3,$$

where C_T and C_Q are constants for any one airscrew, such that $C_T\left(\frac{T}{\rho n^2 D^4}\right)$ and $C_Q\left(\frac{Q}{\rho n^2 D^5}\right)$ are each equal to unity when $(V/np_e)=0\cdot5$, and p_e represents the

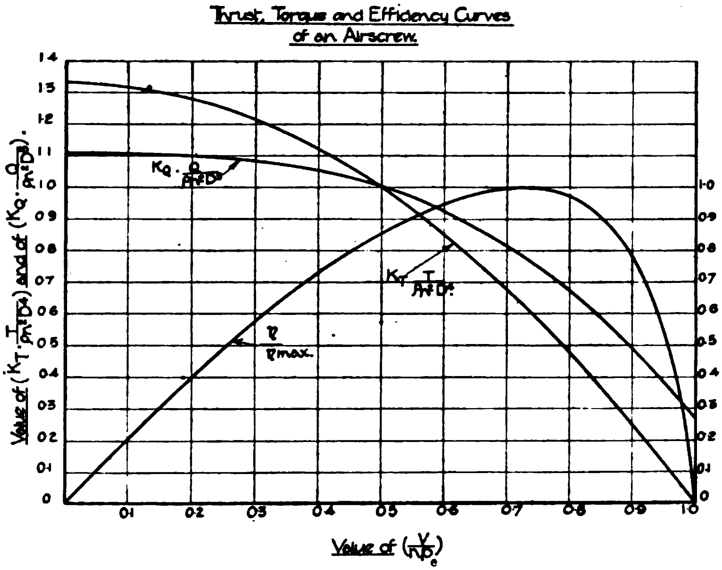


Fig. 34.

experimental mean pitch of the airscrew. The above formulæ were deduced from the performance curves of a large number of model airscrews of widely different characteristics which were tested at the National Physical Laboratory. The formulæ hold with good accuracy—about 5 per cent—over the working range of the 15 airscrews of which the performances were considered. These airscrews were designed for low horse-powers—about 100—and low speeds of translation and rotation—about 90 m.p.h. and 1200 r.p.m. respectively. With high-speed airscrews absorbing large horse-powers it is to be expected that the

* "Notes on the Prediction and Analysis of Aeroplane Performance," by L. Bairstow and J. D. Coales. With Appendix by Miss A. D. Betts, B.SC. Advis. Comm. Aeron., May, 1918.

constants of the formulæ will differ from those given above. From these formulæ it is seen that the efficiency, η,

$$=\frac{p_e}{2\pi D}\cdot\frac{C_Q}{C_T}\cdot\frac{\left[\left(\frac{V}{np_e}\right)-\left(\frac{V}{np_e}\right)^3\right]}{\left[0\cdot828-0\cdot625\left(\frac{V}{np_e}\right)^3\right]}.$$

$$\text{Hence}\qquad \eta_{max.}=0\cdot583\frac{p_e}{2\pi D}\cdot\frac{C_Q}{C_T},$$

and occurs when

$$\left(\frac{V}{np_e}\right)=0\cdot725.\quad \text{Also } \eta=\eta_{max.}\left\{\frac{1\cdot715\left[\left(\frac{V}{np_e}\right)-\left(\frac{V}{np_e}\right)^3\right]}{\left[0\cdot825-0\cdot625\left(\frac{V}{np_e}\right)^3\right]}\right\}.$$

The performance curves calculated from the preceding general equations for thrust, torque, and efficiency are plotted in Fig. 34.

It has been suggested by Soreau* that the performance of an airscrew could be expressed by the equations

$$\left(\frac{T}{n^2}\right)=A+A'\left(\frac{V}{n}\right)+A''\left(\frac{V}{n}\right)^2$$

$$\text{and}\qquad \left(\frac{Q}{n^2}\right)=B+B'\left(\frac{V}{n}\right)+B''\left(\frac{V}{n}\right)^2,$$

$$\text{and also that}\qquad \left(\frac{T}{n^2}\right)=A\left[1-\left(\frac{V}{na'_0}\right)^r\right],$$

$$\text{and}\qquad \left(\frac{Q}{n^2}\right)=B\left[1-\left(\frac{V}{na''_0}\right)^r\right],$$

$$\text{where}\qquad a'_0=\left(\frac{V}{n}\right)_{T=0}\ \text{and}\ a''_0=\left(\frac{V}{n}\right)_{Q=0}.$$

Logarithmic Diagrams and Nomographic Charts

Whilst the curves plotted from the absolute coefficients of thrust, torque, efficiency, and (V/nD) give all the fundamental data measured from experiments, there are of course other methods of presenting the performance of an airscrew. Other absolute coefficients which may be of use in certain circumstances are $(P_e/\rho n^3D^5)$ and $(P_u/\rho n^3D^5)$, where P_e represents the power absorbed by the airscrew and P_u the useful power realised in the forward movement of the aeroplane. The units with which P_e and P_u are measured should of course be dynamically consistent with those of ρ, n, and D.

Two ingenious methods of representing the performance of an airscrew are

* "L'hélice propulsive," Soreau, R. "Mémoires de la Société des Ingénieurs Civils de France," September, 1911.

the logarithmic diagrams of Eiffel* and the nomographic charts of Slocum.† The merit of each of these graphical methods is that for any given *type* of airscrew the performance—or if necessary the diameter and the working speeds needed to develop a given thrust or absorb a given torque—can be directly estimated with good accuracy. With any single airscrew, however, the time and labour involved in the construction of either the logarithmic diagram or the nomographic chart are somewhat excessive. Also with the Polar Logarithmic Diagrams of Eiffel the results obtained partly depend on the determination of the point of intersection of lines which meet at an acute angle and so may be in appreciable error. The method is, however, an interesting application of vector algebra.

With the nomographic charts the performance of an airscrew is expressed firstly in general equations, which are then reduced to a linear form by means of logarithms. Use is then made of logarithmic scales in such a manner that the required data may be obtained by connecting the points representing the given data by straight lines and then reading off the intercepts on the appropriate scales. The method is very similar to that of the slide rule.

Dependence of the Performance of an Airscrew on the Characteristics of the Aeroplane and Engine

Up to the present we have considered the performance of the airscrew regarded as a separate contrivance, but it will now be shown that the working performance

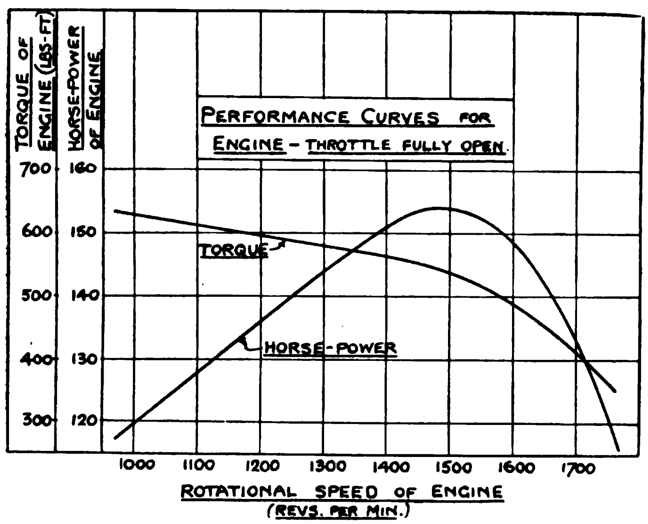


Fig. 35.

of any airscrew depends on the type of engine on which it is mounted. In fact an airscrew can only be efficiently designed when the data of the engine performance and also that of the aeroplane are known.

We shall assume that the airscrew, of which a photograph is given in Fig. 29,

* "Nouvelles recherches sur la résistance de l'air et l'aviation," G. Eiffel.

† "Nomographic charts for the aerial propeller," S. E. Slocum. "Aerial Age," January, 1919.

is driven by an engine of which the characteristics are given in the curves of Fig. 35. It should be stated that the performances of both the aeroplane and the airscrew are calculated for the density of air at ground level. Also it is considered that the engine throttle is fully open and that the airscrew is driven directly from the engine shaft. From the curves of engine performance it will be seen that the torque of the engine decreases as the speed of rotation increases, and also that there is a certain economical rotational speed at which

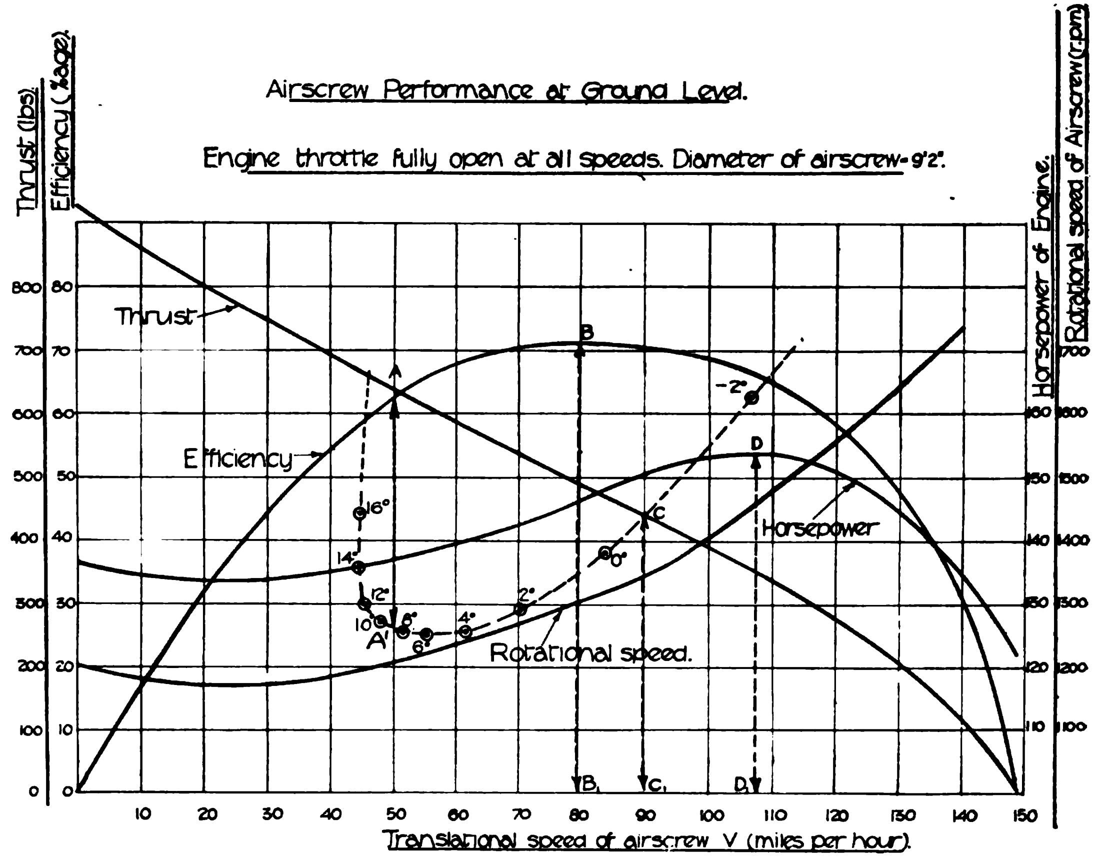


FIG. 36.

the maximum horse-power of the engine is developed. The performance curves of the airscrew at ground level when driven by this engine working all out are given in Fig. 36. It is seen that the maximum thrust of the airscrew is developed when the aeroplane is stationary on the ground, and as the speed of the aeroplane increases the thrust diminishes. It will be shown later that the maximum translational speed of the aeroplane on which this combination of airscrew and engine is assumed to be mounted is 90 m.p.h., the corresponding thrust of the airscrew being 430 lb. If the aeroplane were diving through the air with a translational speed of 150 m.p.h., with the engine working all out, the thrust of the airscrew

would be zero. The distance which the aeroplane moved through the air during one revolution of the airscrew would then be equal to the experimental mean pitch. In such a case the engine would be rotating at an excessively high speed. The rotational speed of either engine or airscrew slightly decreases at first with an increase of the translational speed, but afterwards increases.

The efficiency of the airscrew increases from zero when the translational speed is zero to a maximum value of 72·0 per cent at a forward speed of 80 m.p.h., and then decreases to zero when the airscrew is giving no thrust. The maximum horse-power of the engine is developed at a translational speed of the aeroplane of 108 m.p.h., the corresponding efficiency of the airscrew being 66 per cent.

The dotted curve gives the resistance of the aeroplane in horizontal flight at the corresponding forward speeds. The figures around the curve give the angles of incidence of the wings at the various translational speeds. At the point " C " the thrust of the airscrew with the engine working all out is just equal to the resistance of the machine in horizontal flight, from which it follows that the maximum speed of horizontal flight is 90 m.p.h. For speeds lower than this value the aeroplane will climb if the engine is working all out, the maximum thrust available for climbing being about 380 lb., at a translational speed of 50 m.p.h. At speeds greater than 90 m.p.h. the thrust of the airscrew will be less than the resistance of the aeroplane in horizontal flight, even if the engine be working all out, so that the aeroplane will glide.

Suppose we were designing a scout aeroplane which has, of course, to fly for long periods at its maximum speed. Then the best possible combination of engine, airscrew, and aeroplane would be that in which the engine gave its maximum horse-power and the airscrew its maximum efficiency at the maximum horizontal flight speed of the aeroplane, that is the lines DD_1, CC_1, and BB_1 of Fig. 36 should coincide. On the other hand, if climbing were the first essential, then the lines AA_1, BB_1, and CC_1 should coincide. If a combination of high speed and rapid climb were needed then obviously it would be desirable that the maximum efficiency line BB_1 of the airscrew, and the maximum horse-power line DD_1 of the engine, should lie between the lines AA_1 of maximum climb and CC_1 of maximum speed; and, moreover, the efficiency and horse-power curves should be as flat as possible, so that the efficiency of the airscrew and the horse-power of the engine, both at climbing and at the maximum speed, are not greatly different from their maximum values.

It follows, then, that when designing an airscrew due consideration must be given to the engine on which it is mounted and the type of aeroplane which it is to drive.

CHAPTER VII

ON THE MUTUAL INTERFERENCE OF AN AIRSCREW AND AN AEROPLANE

INTRODUCTION

WITH the present type of aeroplane the airscrew is mounted in close proximity to the engine and the neighbouring parts of the aeroplane, so that to obtain data of direct practical utility it is necessary to measure the performance of an airscrew when working at conditions similar to those of practice. Experiments with an airscrew rotating alone are, however, often of theoretical value with investigations of a special character. A comprehensive series of experiments has been made at the National Physical Laboratory to measure how the performance of an airscrew is modified by the interference of the aeroplane and also how the outflowing or the inflowing stream of the airscrew, as the case may be, affects the resistance of the aeroplane. In the first case the performances of model airscrews were measured both when rotating alone and when rotating in position on model aeroplanes. In the second case the resistances of model aeroplanes were measured (*a*) without the airscrews mounted in place, and (*b*) with the airscrews rotating as in practice. As would be expected, the nature and magnitude of the mutual interference of an airscrew and an aeroplane depend on a large number of parameters : such as the characteristics and position of the airscrew, the horse-power, and the type of engine, and finally the characteristics of the aeroplane. Experiments have been made with both tractor and pusher model aeroplanes and with models of both stationary and rotary engines. No statement of the quantitative effect of mutual interference can be made which would be of sufficient generality to apply with good accuracy to any particular aeroplane. It is necessary to measure separately the mutual interferences of each particular combination of airscrew, engine, and body. For this reason, then, it is proposed in the present chapter to consider in some detail the experimental data of the mutual interferences of several fairly representative types of both airscrew and aeroplane. In the preliminary experiments at the National Physical Laboratory measurements were made of the mutual interference between an airscrew and a part of the aeroplane, such as the body, with one or more of the attached parts, such as landing gear, rudder, elevator, engine fairing, body struts, wings, wing struts. Some later experiments have, however, been made with models of complete aeroplanes.

From a theoretical standpoint it may be considered that the primary effect of the presence of the body is to slow up the air in the neighbourhood of the

airscrew, so that to develop, at the same rotational speed, either the same thrust or the same torque, the forward speed of the airscrew relative to the undisturbed air needs to be increased. It has been shown experimentally that the interference effect is not entirely of a " slowing-up " nature, since the same increase of forward speed is not sufficient to maintain constant both the thrust and the torque. Making a comparison between the performances of the airscrew, with and without the aeroplane interference, and at the same values of the thrust and of the rotational speed it follows that with the body interference the airscrew is doing more useful work, although the increase of efficiency is not so great as would be expected from the increase of the forward speed, because, owing to the increase of the torque, the horse-power absorbed is also greater. It would appear, then, that when estimating the magnitude of the effect of the aeroplane interference on the performance of an airscrew, account should be taken of the engine performance with which the relationship between the torque and the rotational speed is definitely fixed. Also, with interference, the airscrew will need to run slower to absorb the same torque at the same forward speed of the machine, so that if the airscrew is designed without any consideration of the body interference it may not be possible to develop the maximum horse-power of the engine.

These points, which are considered in detail later, indicate that it is of importance that the combination of airscrew and engine should be considered when estimating the interference effect.

We shall now consider in some detail the mutual interferences of the airscrew and the body of several representative aeroplanes with stationary engines.

The airscrew nomenclature used in the present chapter is the same as hitherto. The new symbols R and R_0 are introduced,

where R_0 = the resistance of the aeroplane—or part of the aeroplane, as the case may be—without the airscrew,

and R = the resistance of the aeroplane—or part of the aeroplane—at the same forward speed at which R was measured, but with the airscrew *detached* from but rotating as in practice.

Hence (R/R_0) represents the ratio of the resistance of the aeroplane—or part of the aeroplane—with the airscrew rotating but detached from the body to the resistance without any airscrew interference.

Mutual Interference of the Airscrew and Body of the Tractor Aeroplane B.E. 2c*

This was the first series of " mutual-interference " experiments made at the National Physical Laboratory. Measurements were made of the mutual interference of the airscrew and the body of the aeroplane. Sketches of both the airscrew and the body are shown in Figs. 37 and 38 respectively. The diameter of full-scale airscrew was 9 ft. 6 in. It will be noticed from Fig. 38 that the model body, of scale one-sixth, as used in the experiments, had no rudder, fin, engine cowling, etc.

* " An investigation of the mutual interference of an airscrew and body of the ' tractor ' type of aeroplane," by A. Fage, A.R.C.SC., and H. E. Collins. Advis. Comm. Aeron., August, 1917.

The results of the experiments with the airscrew are shown graphically in Fig. 39. The maximum efficiency is increased by 4 per cent, that is from 75·5 per cent to 78·5 per cent, by the interference of the body.

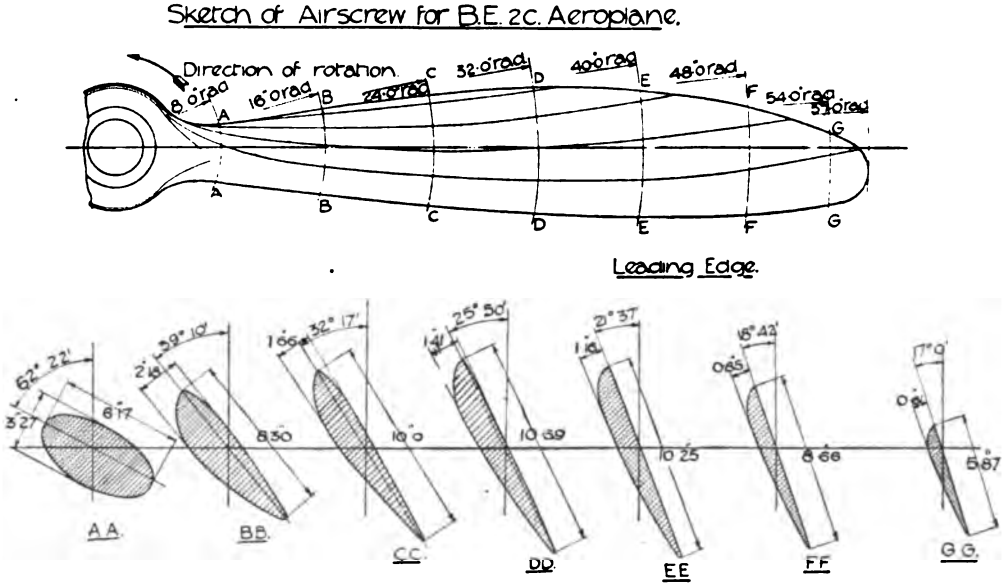


Fig. 37.

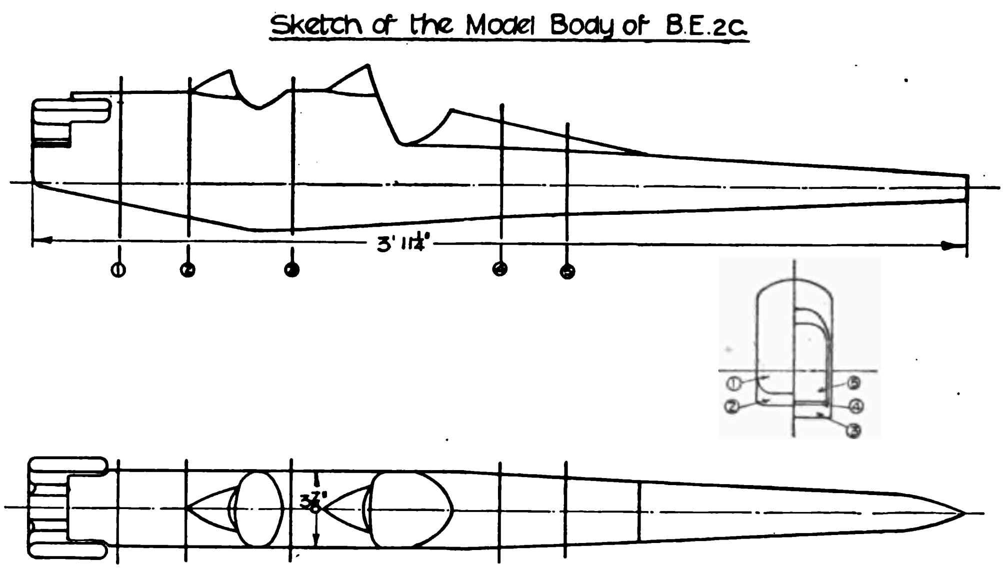


Fig. 38.

At the climb the efficiency is increased by about 1 or 2 per cent. At any given values of the translational and rotational speeds of the airscrew, both the thrust and the torque are increased by the presence of the body. Generally

speaking, to develop the same thrust and torque at the same rotational speed, the translational speed of the airscrew relatively to undisturbed air has to be increased about 3 to 5 per cent, the percentage increase being smaller at the higher value of (V/nD).

The resistance of the model body as measured at a wind speed of 40 ft. per sec. was 0·1455 lb. The resistance at a wind speed of 100 ft. per sec. of a similar full-scale body—without cowling, etc.—as calculated from the experimental data is therefore 32·8 lb. It will be seen from the curve of Fig. 40 that the relationship

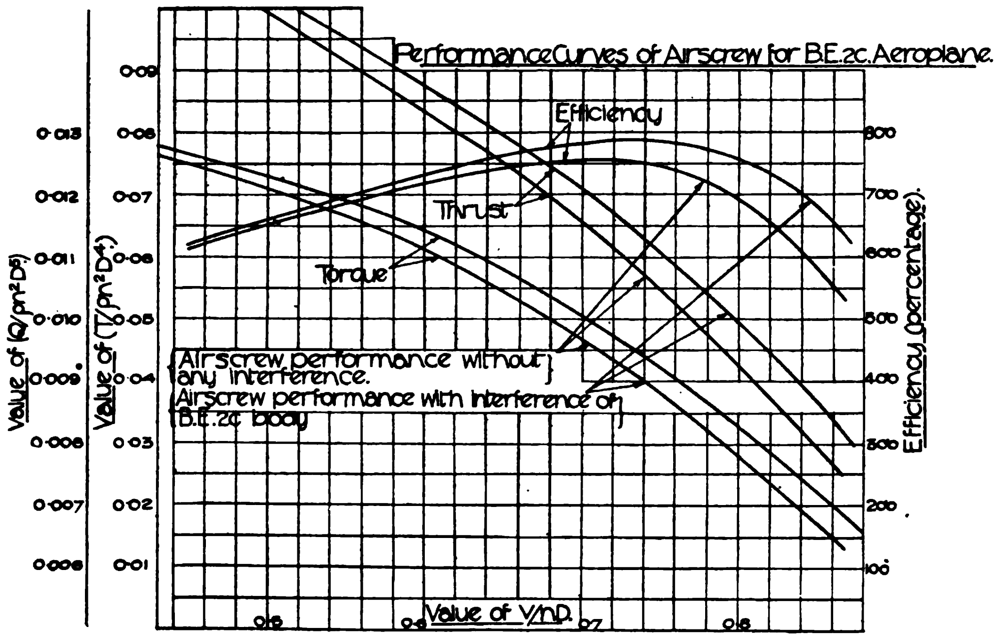


FIG. 39.

between (R/R_0) and $(T/\rho V^2D^2)$ of the airscrew working with the interference of the body, may be expressed approximately by the linear equation $(R/R_0)=0{\cdot}87+2{\cdot}18(T/\rho V^2D^2)$. As previously defined, (R/R_0) is the ratio of the resistance coefficient of the body with interference from the airscrew, to the resistance coefficient of the body without interference.

It is of some theoretical interest to compare the value of (R/R_0) as measured experimentally with the value calculated from the "Froude" momentum theory. We have already seen that with this theory $T=\frac{\pi D^2}{4}\rho(1+a)2V^2a$, where $2aV$ is the velocity of the outflowing stream measured relative to undisturbed air.

Also since the body is entirely enclosed within this outflowing stream we may write

$$(R/R_0)=(1+2a)^2.$$

Hence $T=\frac{\pi D^2}{8}\rho.V^2.(R/R_0-1)$,

that is, $(R/R_0)=1{\cdot}0+2{\cdot}55(T/\rho D^2V^2)$.

A G

The curve given by this equation is plotted in Fig. 40, from which it will be seen that at any value of the thrust, the theoretical calculation overestimates the value of (R/R_C). This is because the velocity of the outflowing stream at

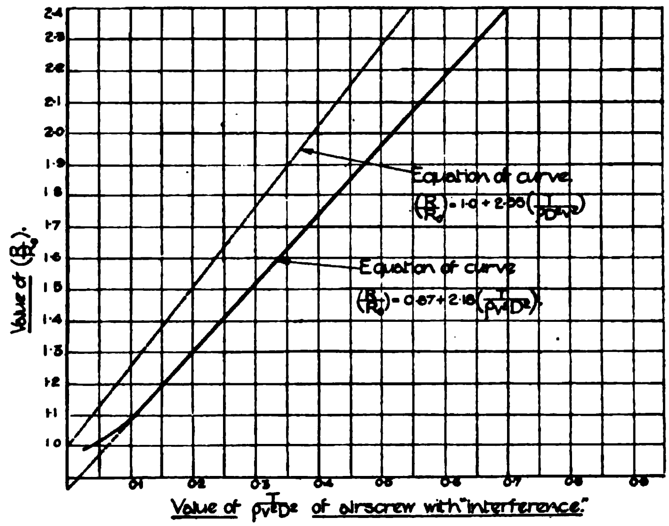


FIG. 40.

the region where the body is placed is probably lower than the calculated value, and also the body is to some extent shielded by the boss of the airscrew.

MUTUAL INTERFERENCE OF THE AIRSCREW, BODY, AND WINGS OF THE TRACTOR AEROPLANE B.E. 2e*

These experiments were made to measure the mutual interference of the airscrew, body, and wings of the tractor aeroplane B.E. 2e. A sketch of the airscrew, which has four blades and a diameter of 9 ft. 1 in., is shown in Fig. 41. A sketch of the model of the body and the wings is given in Fig. 42. A representation of the R.A.F. engine with cowling was fitted at the front of the body. The landing gear was attached to the body, but the rudder, elevator, and fin were not mounted. The model wings were of section R.A.F. 14 with struts in place. The scale of the models was one-sixth full size. Experiments were made to find the interference on the performance of the airscrew, (*a*) of the body alone, and (*b*) of the combination of body and wings.

The results of the airscrew experiments are shown graphically in Figs. 43–45. As in the previous experiments, the presence of the body and wings slows up the air in the neighbourhood of the airscrew, resulting, as shown in Figs. 43 and 44, in a shift of the thrust and torque curves parallel to the axis of (V/nD). To develop the same thrust at the same rotational speed, the translational speed of the airscrew relatively to the undisturbed air needs to be increased by about

* "An investigation of the mutual interference of the airscrew, body, and wings of the tractor aeroplane B.E. 2e.," by A. Fage, A.R.C.SC., and H. E. Collins. Advis. Comm. Aeron., 1918.

Airscrew for B.E.2e Aeroplane.

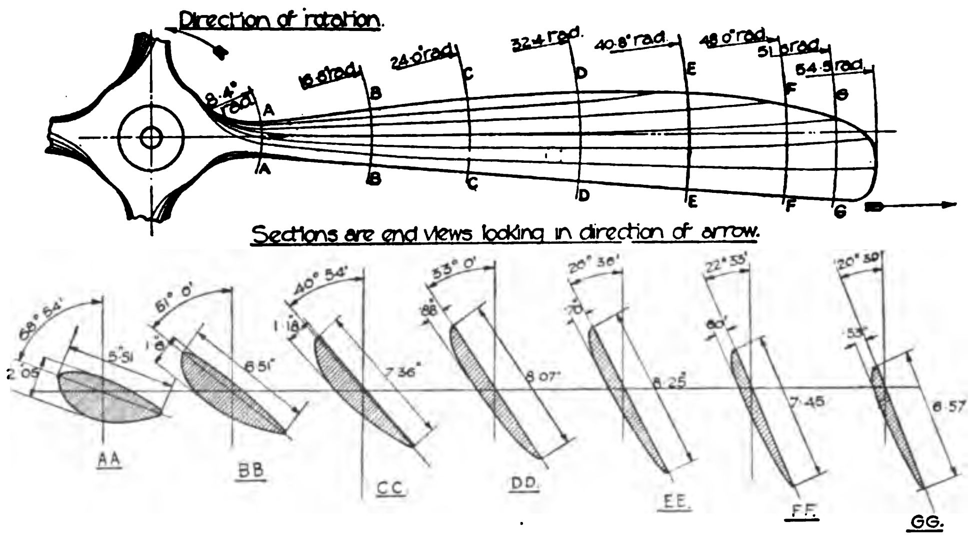


FIG. 41.

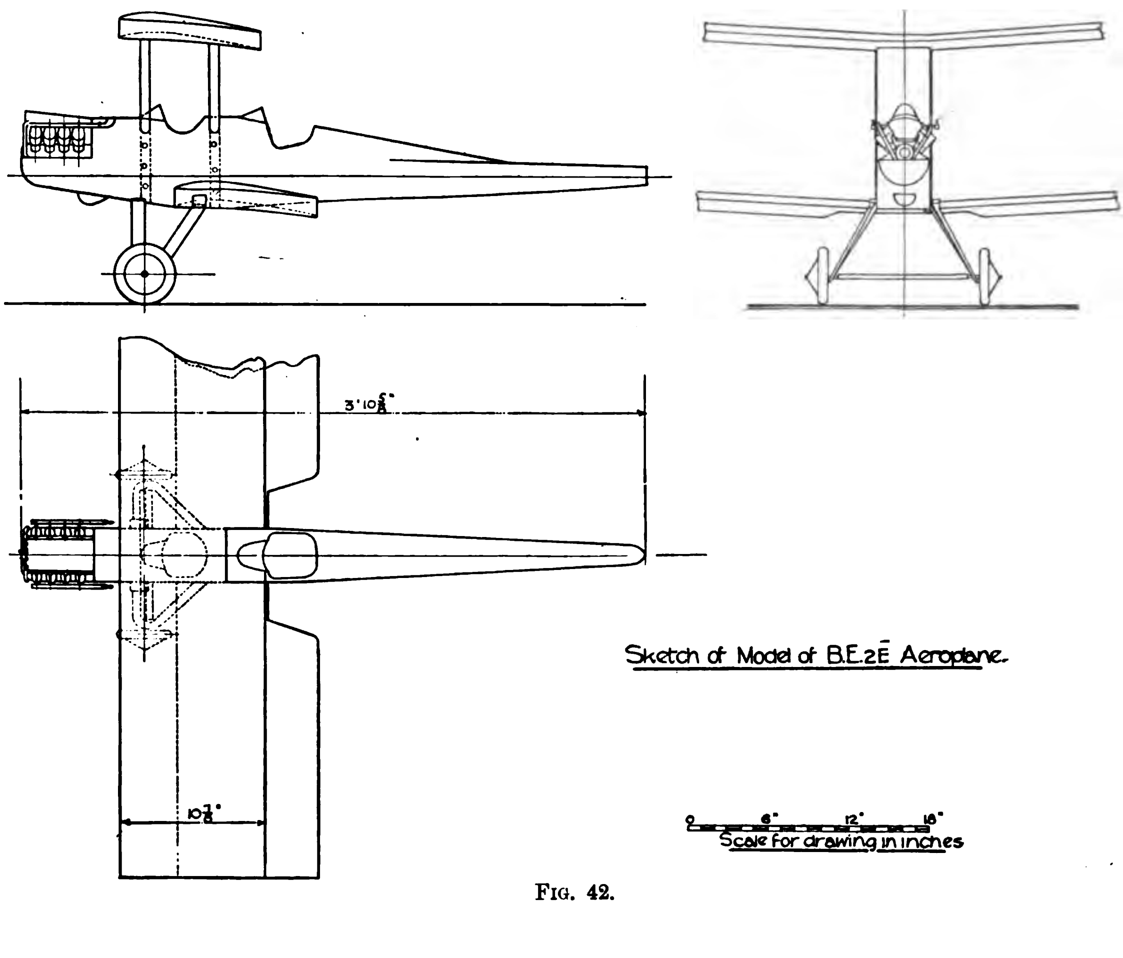


FIG. 42.

4·0 per cent due to the presence of the body alone, and about 5·0 per cent due to the combined interference of the body and wings. The effect of the interference of the body is to increase, at any value of (V/nD), the value of the torque coefficient by about 0·0018, or expressing the interference in a similar manner as with the thrust, the same torque will be developed at the same rotational speed, when the body is behind the airscrew, if the translational speed be increased by 3·0 per cent at high values of (V/nD) and about 8·0 per cent when climbing. Within the limits of accuracy of the experiments, the combined interference of the body and wings on the torque is the same as that due to the body alone.

The values of the maximum efficiency of the airscrew alone, of the airscrew with the interference of the body, and of the airscrew with the combined inter-

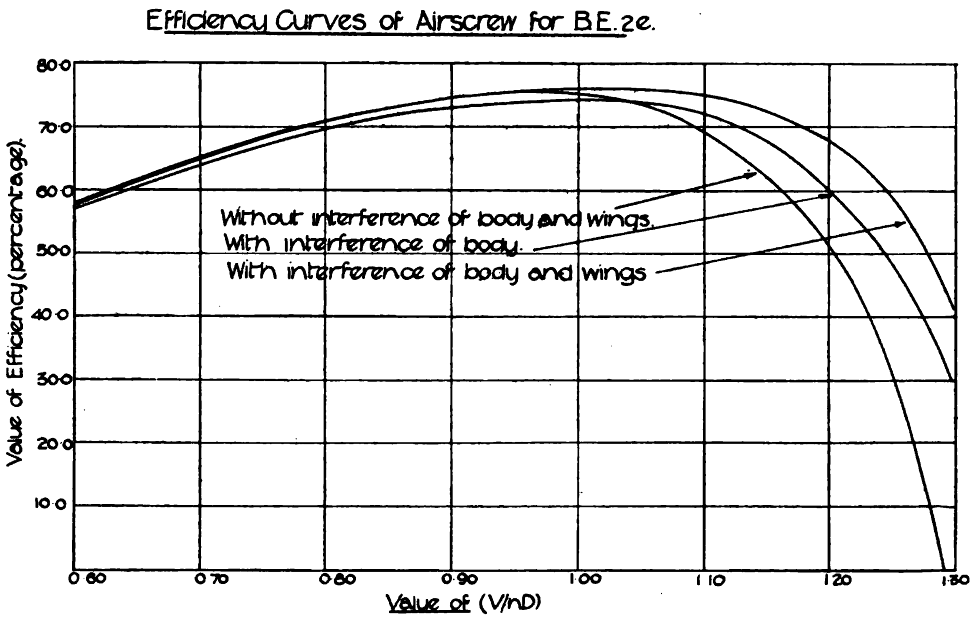


Fig. 43.

ference of body and wings are 75·5 per cent, 74·0 per cent, and 76·0 per cent respectively. At climbing speeds, the efficiency of the airscrew with the interference of the body alone is about 2 per cent lower than that of the airscrew without any interference. The efficiency at the lower values of (V/nD) is practically unaffected by the combined interference of the body and wings.

We found that at a wind speed of 40 ft. per sec. the resistance of the model body alone (see Fig. 42) was 0·34 lb. At the same wind speed, the resistance of the model body with the interference of wings and struts—the wings and struts were supported in place and not attached to the body—was 0·365 lb. Hence at a wind speed of 100 ft. per sec. the resistances of a similar full-scale body alone, and also of a similar full-scale body with interference of wings and struts, would be 76·5 lb. and 82 lb. respectively.

As in the previous experiments, (R/R_0) was a linear function of $(T/\rho V^2D^2)$.

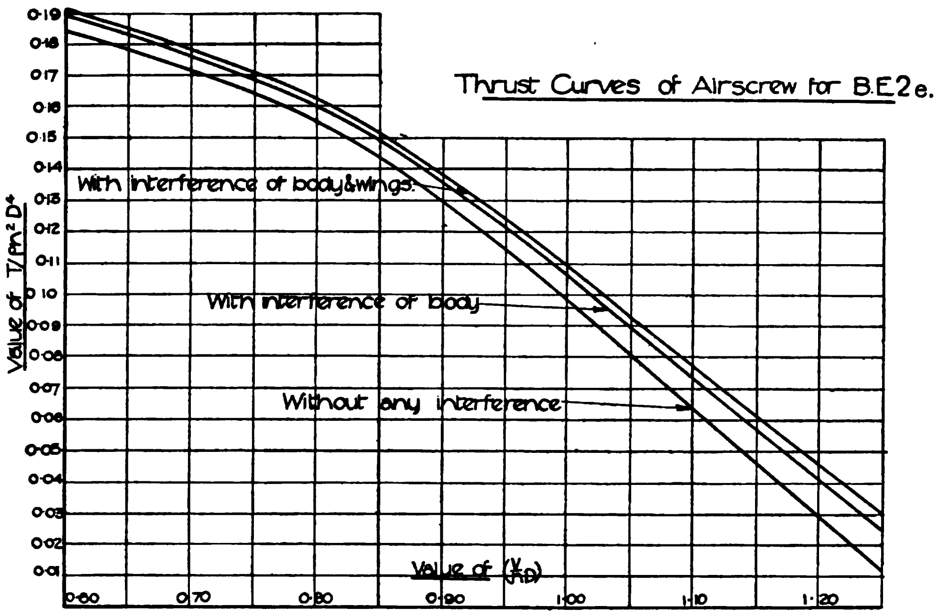

Thrust Curves of Airscrew for B.E.2e.
With interference of body & wings.
With interference of body
Without any interference
Value of T/ρn²D⁴
Value of (V/nD)
0·19
0·18
0·17
0·16
0·15
0·14
0·13
0·12
0·11
0·10
0·09
0·08
0·07
0·06
0·05
0·04
0·03
0·02
0·01
0·60
0·70
0·80
0·90
1·00
1·10
1·20

FIG. 44.

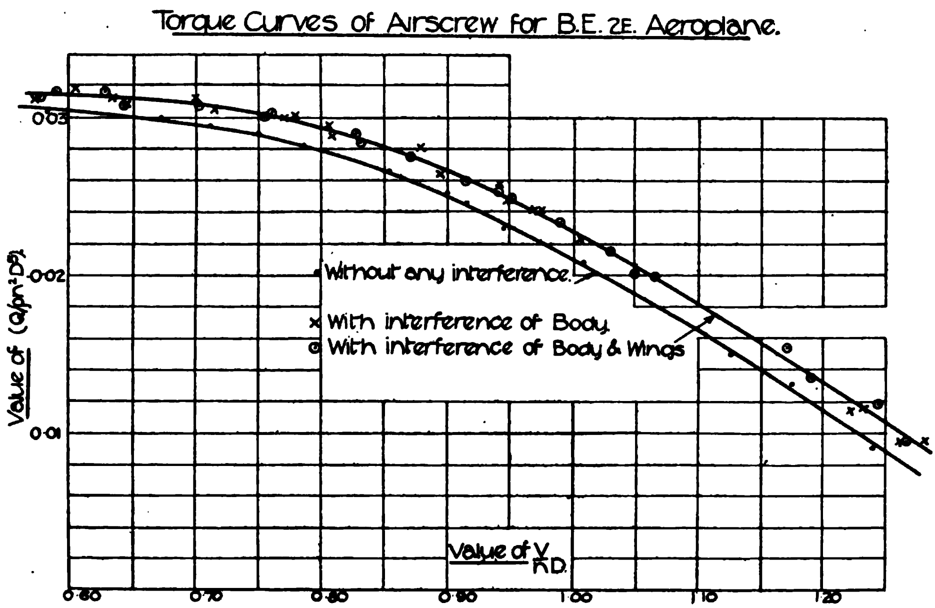

Torque Curves of Airscrew for B.E. 2E. Aeroplane.
Without any interference.
× With interference of Body.
⊙ With interference of Body & Wings
Value of (Q/ρn²D⁵)
Value of V/nD.
0·03
0·02
0·01
0·60
0·70
0·80
0·90
1·00
1·10
1·20

FIG. 45.

With no interference from the wings we found that $(R/R_0) = 0{\cdot}855 + 1{\cdot}32(T/\rho V^2D^2)$, where $(T/\rho V^2D^2)$ is the absolute thrust coefficient of the airscrew working with the interference of the body alone. With the interference from both wings and body, $(R/R_0) = 0{\cdot}83 + 1{\cdot}18(T/\rho V^2D^2)$. In the latter case $(T/\rho V^2D^2)$ is the absolute thrust coefficient of the airscrew working with the interference of both body and wings. The value of (R/R_0), which would be calculated from the theoretical equation $(R/R_0 = 1{\cdot}0 + 2{\cdot}55(T/\rho V^2D^2))$, considerably overestimates, therefore, the experimental value in both these cases. Whilst it is probable that the velocity of the air in the neighbourhood of the body is lower than that calculated, it is expected that the large part of the discrepancy between the theoretical and experimental resistances is because of the "blocking up" of the front of the body by the boss of the airscrew and also because a part of the landing gear is without the outflowing stream from the airscrew.

The resistance of the body alone has a high value owing to the flow of air round the engine and through the cowling, and the conclusion which may be drawn from the interference experiments is that the air-flow at the front of the body is appreciably modified by the working of the airscrew. It would seem that up to quite moderate values of the thrust a smaller quantity of air is flowing over the engine and through the cowling than when the airscrew is not running. At the same value of $(T/\rho V^2D^2)$ the value of (R/R_0) is reduced by the interference of wings, the wings influencing both the speed and direction of the outflowing stream, but they probably do not appreciably interfere with the flow of air through the cowling.

Mutual Interference of the Airscrew and the Body of the Pusher Aeroplane F.E. 2b.*

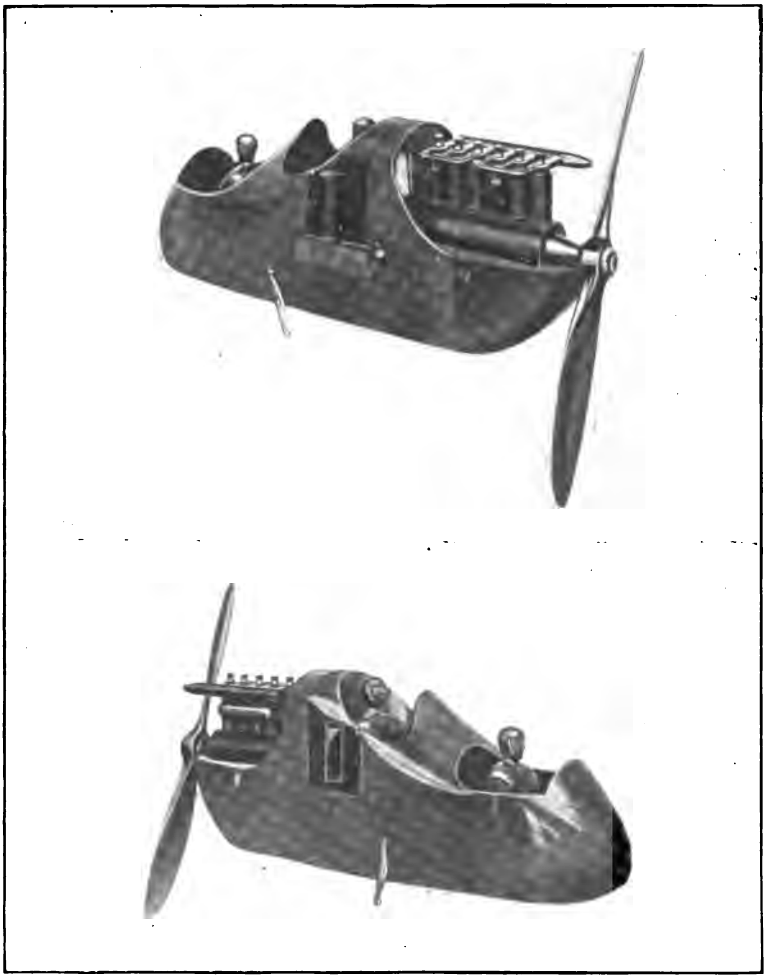
These were the first experiments made to measure the mutual interference of an airscrew and body of an aeroplane of the pusher type. The airscrew used was very similar to the two-bladed airscrew designed for the aeroplane R.E. 7. A photograph of the full-scale airscrew, of which the diameter is 9 ft. 2 in., is shown in Fig. 29.

The model airscrew was mounted behind a model of the 120 h.p. Austro-Daimler engine, the radiator of the engine being represented with good accuracy by two sheets of wire gauze of 6 meshes to the inch and 24 S.W.G. Photographs of the model body of the F.E. 2b machine with the airscrew mounted in place are shown in Figs. 46 and 47. The data of the airscrew experiments are shown in Figs. 48 and 49. It will be noticed that at the same value of (V/nD) both the thrust and the torque coefficients are increased by the presence of the body. Taking an average over the working range of (V/nD), it will be seen that the same thrust will be developed at the same rotational speed, with the body in position, if the translational speed of the airscrew relative to undisturbed air be increased by about 10 per cent. From Fig. 49 it is seen that the presence of the body

* "An investigation of the mutual interference of airscrew and bodies of the 'pusher' type," by A. Fage, A.R.C.SC., and H. E. Collins. Advis. Comm. Aeron., 1917.

increases the maximum efficiency of the airscrew from 67·5 per cent to 72 per cent. We also found that the relationship between (R/R_0) and $(T/\rho V^2D^2)$ may be expressed by the equation

$$R/R_0=0{\cdot}965+1{\cdot}06(T/\rho V^2D^2).$$



FIGS. 46 AND 47.

The absolute thrust coefficient $(T/\rho V^2D^2)$ includes, as formerly, the interference of the body on the airscrew. The resistance of the model of the F.E. 2b body as measured at a wind speed of 50 ft.-sec. was found to be 0·425 lb. Since the scale of the model was one-sixth, the resistance of a similar full-scale body at a wind speed of 100 ft.-sec. would be 61 lb.

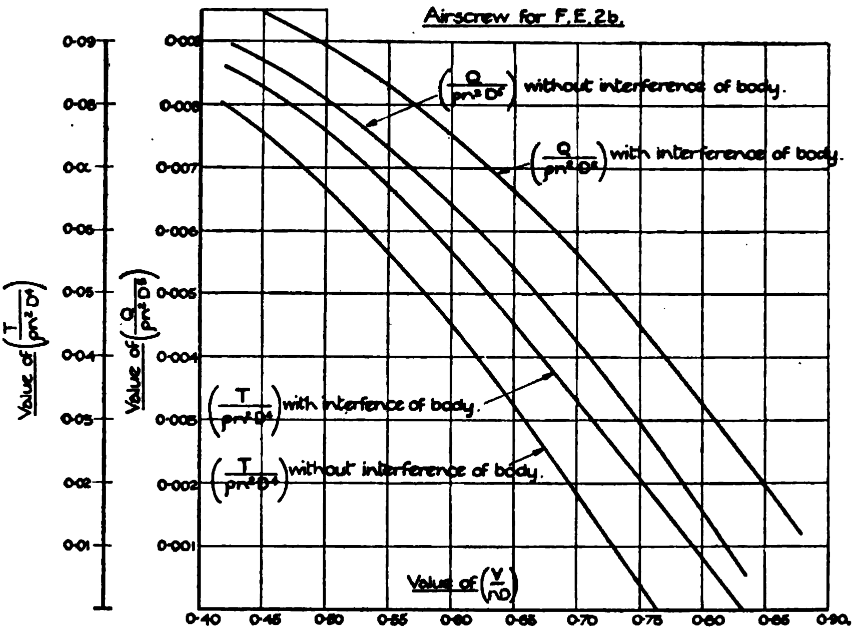

Airscrew for F.E.2b.
(Q/ρn²D⁵) without interference of body.
(Q/ρn²D⁵) with interference of body.
(T/ρn²D⁴) with interference of body.
(T/ρn²D⁴) without interference of body.
Value of (T/ρn²D⁴)
Value of (Q/ρn²D⁵)
Value of (V/nD)
0·40
0·45
0·50
0·55
0·60
0·65
0·70
0·75
0·80
0·85
0·90
0·01
0·02
0·03
0·04
0·05
0·06
0·08
0·09
0·001
0·002
0·003
0·004
0·005
0·006
0·007
0·008

FIG. 48.

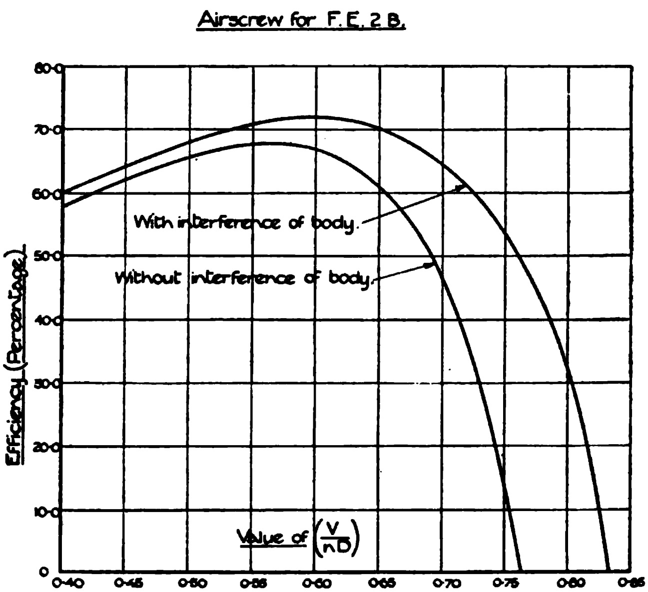

Airscrew for F.E.2B.
With interference of body.
Without interference of body.
Efficiency (Percentage).
Value of (V/nD)
0
10·0
20·0
30·0
40·0
50·0
60·0
70·0
80·0
0·40
0·45
0·50
0·55
0·60
0·65
0·70
0·75
0·80
0·85

FIG. 49.

Mutual Interferences of Airscrews and a Tractor Body of Irregular Shape*

Mention should be made of a series of experiments made at the National Physical Laboratory to measure the mutual interferences of five model airscrews and a model of the end of the whirling arm at the Royal Aircraft Establishment. As far as the present discussion is concerned, the model of the end of the R.A.E. whirling arm may be considered to represent an aeroplane body of large size and of irregular shape. We found that the interferences of this body, which were more pronounced than the interference of an ordinary aeroplane body, were different with airscrews of different characteristics.

An Analysis of the Mutual Interferences of Aeroplane Bodies and Airscrews†

It is not possible to obtain a clear conception of the nature of the mutual interference of an airscrew and body, if the airscrew data are presented in terms of absolute coefficients. Consideration must also be taken of the performance of the engine.

Accordingly, then, the following analysis of the preceding experimental data is made to obtain some notions of both the physical and numerical effects of the mutual interferences of aeroplane bodies—with stationary engines—and airscrews. The method of the analysis is to calculate the performance of each airscrew, when mounted on the engine for which it was designed, both with and without the body interference. It is, of course, not necessary to use the actual horse-power curve of each engine. A constant torque as calculated from the maximum horse-power of the engine may be assumed over the working range of the rotational speed.

Calculations are made for four combinations of airscrew and aeroplane, namely :—

(1) Airscrew and body of tractor machine, B.E. 2c. Landing gear, wings, and engine cowling were not attached to the body. See p. 79.

(2) Airscrew and body of tractor machine, B.E. 2e. Landing gear and engine cowling were attached to the body. See p. 82.

(3) Airscrew and body of pusher machine F.E. 2b. See p. 86.

(4) A tractor body equivalent in its interference effect to that of the R.A.E. whirling arm and an airscrew which may be regarded as having the mean performance of four airscrews of different characteristics. See p. 89.

The data of such calculations are collected in Table VI.

* "Tests on five model airscrews and an experimental investigation of the interferences between these airscrews and a model of the end of the whirling arm at the Royal Aircraft Establishment," by A. Fage, A.R.C.SC., and H. E. Collins. Advis. Comm. Aeron., 1916.

† "An analysis of the mutual interferences of aeroplane bodies and airscrews," by A. Fage, A.R.C.SC., and H. E. Collins. Advis. Comm. Aeron., 1918.

TABLE VI

Calculations were made for the density of the air at ground level.
Torque of engine assumed constant over the working range of forward speed.

T = Thrust of airscrew alone.
T_1 = Thrust of airscrew with body interference.
n = Rotational speed of airscrew alone.
n_1 = Rotational speed with body interference.
η = Efficiency of airscrew alone.
η_1 = Efficiency of airscrew with body interference.
R = Resistance of body with airscrew working.
R_0 = Resistance of body alone, without the airscrew in place.

Description of airscrew, body, etc.	Forward speed miles/hr.	Value of (T_1/T)	Value of (n_1/n)	Value of (η_1/η)	Value of $\left(\frac{T_1-(R-R_0)}{T}\right)$
Airscrew for pusher machine F.E. 2b. Diameter of airscrew 9 ft. 2 in. Airscrew assumed to be mounted on 160 h.p. Beardmore engine. Torque of airscrew taken as 710 lb.-ft.	90	1·020	0·960	1·060	0·994
	85	1·020	0·965	1·055	0·988
	80	1·015	0·966	1·055	0·983
	70	1·010	0·967	1·050	0·973
	60	1·010	0·970	1·045	0·970
	50	1·005	0·975	1·035	0·966
Airscrew for B.E. 2e machine. Diameter of airscrew 9 ft. 1 in. Airscrew assumed to be mounted on R.A.F. 1A engine. Torque of airscrew taken as 585 lb.-ft. Body had landing gear and engine cowling.	90	1·020	0·980	1·040	1·037
	80	1·010	0·982	1·025	1·005
	70	1·000	0·982	1·015	0·970
	60	1·000	0·984	1·015	0·958
	50	1·000	0·986	1·010	0·952
Airscrew for B.E. 2c machine. Diameter of airscrew 9 ft. 6 in. Airscrew mounted on a 70 h.p. Renault engine. No landing gear, no engine cowling. Torque of airscrew taken as 560 lb.-ft.	90	1·050	0·985	1·065	1·040
	80	1·035	0·987	1·045	1·018
	70	1·015	0·987	1·030	0·997
	60	1·010	0·990	1·020	0·980
	50	1·C05	0·992	1·015	0·968
A hypothetical airscrew having the "mean" interference effect of four airscrews, and a hypothetical body equivalent in its interference effect to that of the R.A.E. whirling arm.	110	1·090	0·968	1·125	Not measured.
	100	1·084	0·973	1·115	
	90	1·080	0·978	1·105	
	80	1·073	0·983	1·090	
	70	1·064	0·988	1·076	
	60	1·053	0·993	1·060	
	50	1·038	0·997	1·040	

From the curves of Fig. 50, which are plotted from the data of this table, it will be seen that with the combinations under consideration, the interference of a body on an airscrew is of the same general nature, that is at any forward speed of the aeroplane, body interference increases both the thrust and efficiency and reduces the rotational speed of the airscrew and engine. Also the interference increases with an increase of the forward speed of the machine. As would be expected, the interferences are most pronounced with the body equivalent to the end of the R.A.E. whirling arm. The interference of a "pusher" body on an airscrew appears to be somewhat greater than that of a "tractor" body, but there is no evidence as to whether this is due to the characteristics of the airscrew or to the position of the body. A general statement of the magnitude of the modification of the performance of an airscrew due to the presence of a body may be made from the data of Fig. 50. It would seem that at the maximum horizontal flight speed of the machine both the thrust and the efficiency are increased by about 3 per cent and 6 per cent respectively, and the rotational speed is decreased by about 3 per cent. When climbing, the interference effect is of smaller magnitude, the thrust and efficiency being increased by 1 per cent and 2·5 per cent, and the rotational speed being decreased by 1·5 per cent. A matter of some importance is immediately obvious from the preceding analysis. If an airscrew were designed for maximum horizontal flight,—without considering body interference—with such nicety as to run at the rotational speed which develops the maximum horse-power of the engine, then both the thrust and efficiency will be greater than the design values when the airscrew is on the aeroplane, but the airscrew will be running slower, so that the engine cannot develop its maximum horse-power. From the data of the investigation it follows, however, that the maximum horse-power of the engine would be developed at

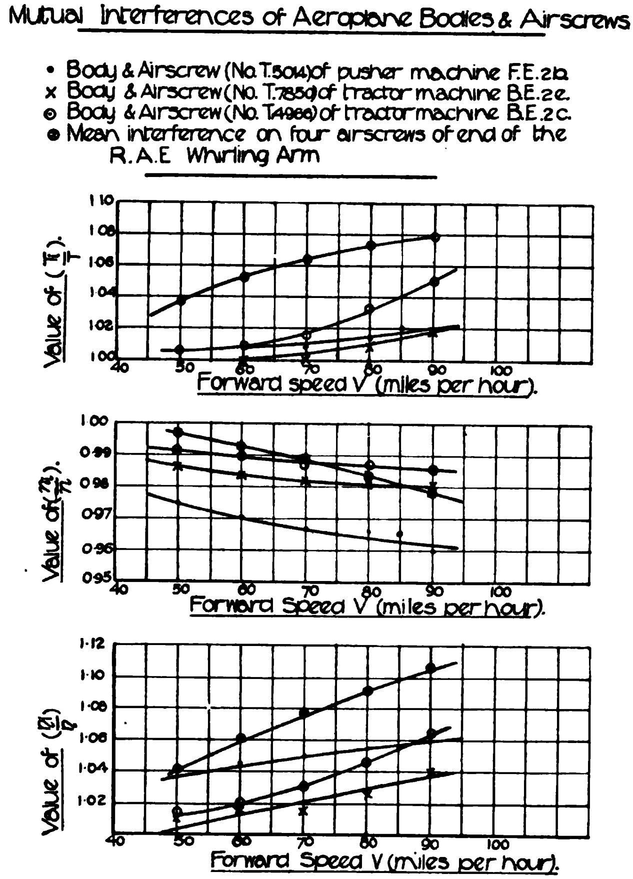


FIG. 50.

the maximum horizontal flight speed of the aeroplane if the rotational speed of the design had been about 3 per cent greater than that at which the maximum horse-power of the engine is developed.

Up to the present the interference of the body on the airscrew has been considered. It is now proposed to consider the effect of the airscrew on the body and also the effect of the mutual interference on the combination of airscrew, engine, and body. From the data of the experiments with the combinations F.E. 2b, B.E. 2c, and B.E. 2e the extra resistances $(R - R_0)$ of each body due to the working of the airscrew at various forward speeds of the machine have been calculated. In the last column of Table VI are given the values of $\left(\frac{T_1 - (R - R_0)}{T}\right)$ at several forward speeds of each machine, where T_1 represents the thrust of an airscrew as calculated from experiments involving mutual interference, and T represents the thrust of the airscrew without the presence of the body. The data of this column have been plotted in Fig. 51, from which it is seen that over the greater part of the speed range the value of $\left(\frac{T_1 - (R - R_0)}{T}\right)$ for each combination is less than unity, that is $(R - R_0)$ is greater than $(T_1 - T)$. It follows, then, that over the greater part of the speed range, the extra resistance of the body due to the interference of the airscrew is greater than the increase of the thrust of the airscrew due to the interference of the body. It would seem, then, that at the lower speeds of flight the performance of the combination of airscrew, engine, and body is disadvantageously affected by the mutual interference of the body and airscrew.

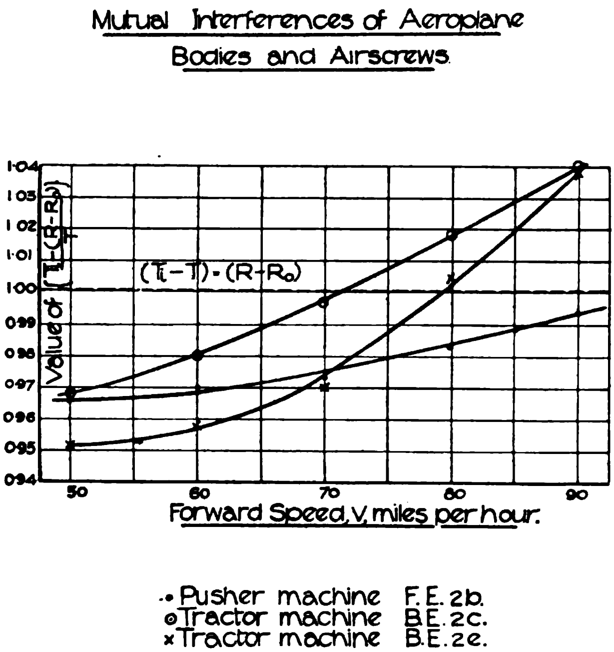


FIG. 51.

Interference of an Airscrew on the Resistance of a Complete Aeroplane

It is now proposed to describe in some detail two series of experiments made with models of complete aeroplanes, namely (*a*) the Sopwith Dolphin Scout and (*b*) the S.E. 5 aeroplane. With both aeroplanes measurements were made of

the performances of model airscrews rotating in front of the model aeroplane, and also of the dependence on the thrust of the resistance of that part of the model aeroplane which is contained within the outflowing stream from each airscrew.

(a) *Sopwith Dolphin Scout Aeroplane**

These experiments were made to measure the performance of two model airscrews when rotating in front of a model of the Dolphin Scout aeroplane and also to measure the interference of each airscrew on the resistance of the aeroplane. Only the data of the experiments made with the model of one-third scale of the two-bladed airscrew A.B. 8080 of diameter 8 ft. 3 in. will be now described. A sketch of this airscrew which was designed for the 200 h.p. Hispano-Suiza engine is shown in Fig. 52.

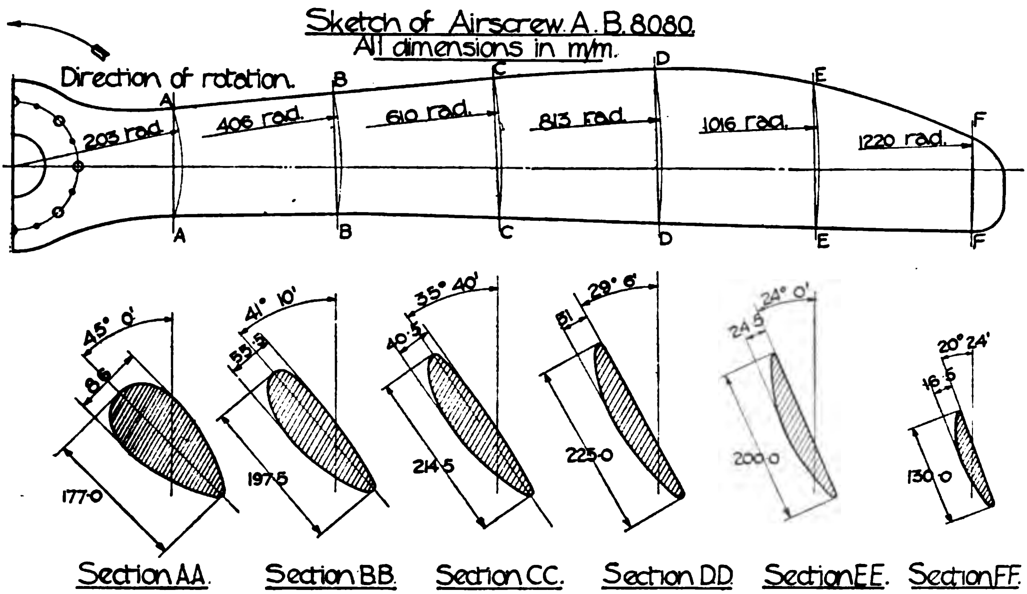


FIG. 52.

An idea of the general shape of the model aeroplane is given by the photograph of Fig. 53—of the general arrangement of the models during the experiments. It will be seen that the body, landing gear, tail plane, wings, wing struts, and skid were represented. Measurements were made of the performances of the airscrew (*a*) with the interference of the body, landing gear, and tail plane; and (*b*) with the interference of the body, landing gear, tail plane, and wings. The data of the airscrew experiments are shown in Fig. 54, from which it will be seen that the airscrew has a maximum efficiency of about 82·5 per cent, and also a high efficiency at the climb. It would appear from the curves of Fig. 54 that the presence of the wings increases slightly both the thrust and the torque, the numerical value of the increase being about 1·5 per cent in each case.

* "An investigation of the mutual interferences of two model airscrews and a model of the Sopwith Dolphin aeroplane," by A. Fage, A.R.C.SC., and H. E. Collins. Advis. Comm. Aeron., 1919.

Measurements were also made, both with and without the airscrew rotating, of the resistances (*a*) of the body with the landing gear and the tail plane, and (*b*) of the body, with the landing gear, tail plane, and those parts of the wings which would be in the outflowing stream. In the latter case the wings were cut at the boundary of the outflowing stream, one portion of each severed wing being attached to the body and the other held in position in such a manner that the continuity of each wing surface was only broken by a narrow slit of width about $\frac{1}{8}$ in.

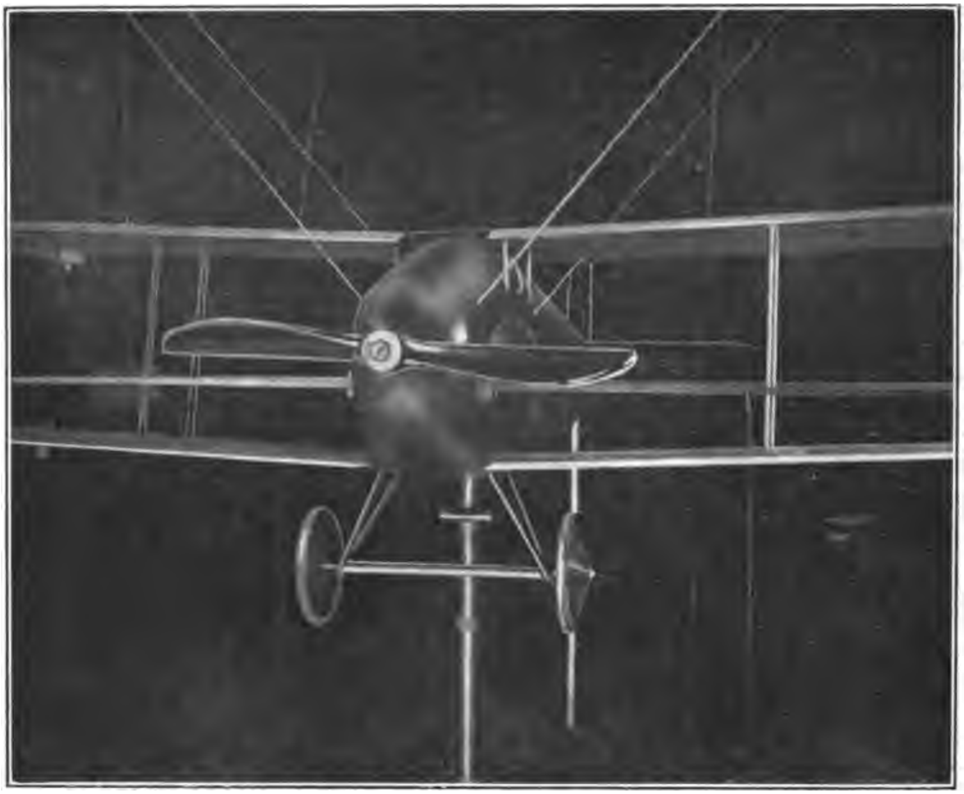


FIG. 53.

In the first case the relationship between (R/R_0) and $(T/\rho V^2 D^2)$ may be expressed by the linear equation $(R/R_0) = 0{\cdot}90 + 3{\cdot}03(T/\rho V^2 D^2)$. In the second case, that is with the inclusion of those parts of the wings which would be in the outflowing stream, $(R/R_0) = 0{\cdot}93 + 3{\cdot}03(T/\rho V^2 D^2)$.

The resistance in a wind of 40 ft. per sec. of the model of the body, landing gear, and tail plane—one-third scale—as measured was 0·77 lb. approximately. At the same speed the resistance of the model of the body, landing gear, tail plane, and the parts of the wings attached to the body—that is those parts which would be in the outflowing stream—was 1·105 lb.

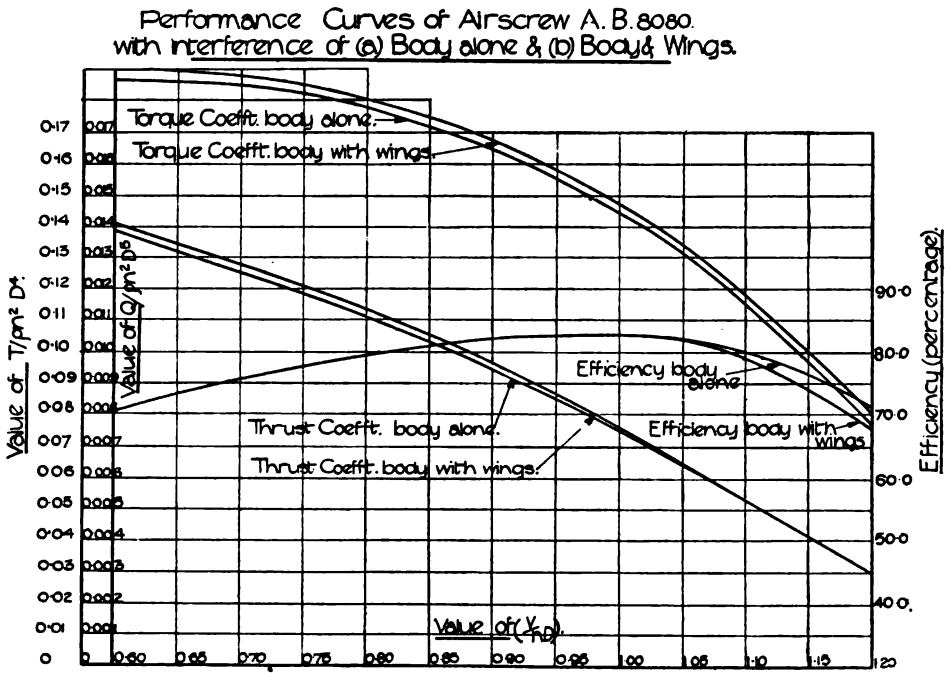

Performance Curves of Airscrew A. B. 8080.
with interference of (a) Body alone & (b) Body & Wings.
Torque Coefft. body alone.
Torque Coefft. body with wings.
Efficiency body alone.
Efficiency body with wings.
Thrust Coefft. body alone.
Thrust Coefft. body with wings.
Value of T/ρn² D⁴.
Value of Q/ρn² D⁵
Efficiency (percentage).
Value of (V/nD).

FIG. 54.

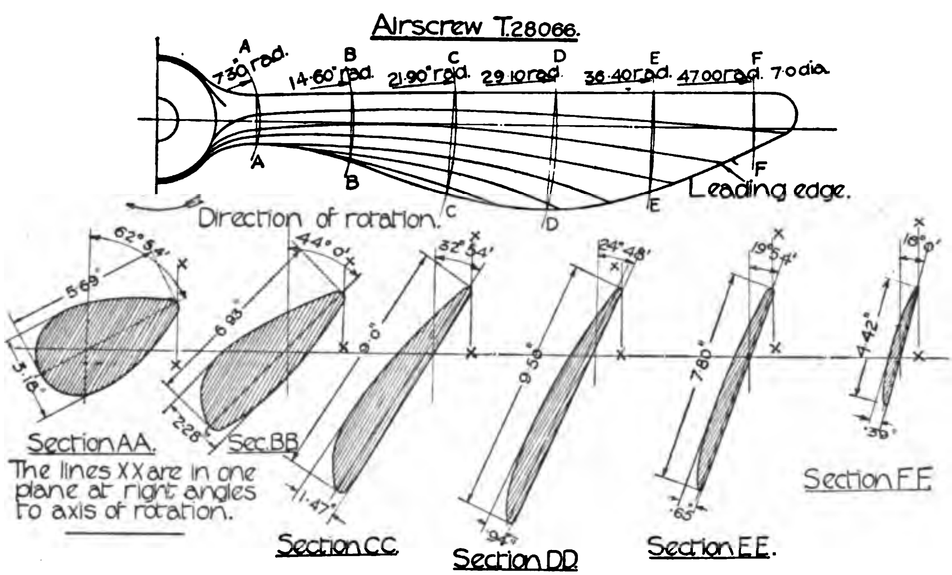

Airscrew T.28066.
Direction of rotation.
Leading edge.
Section AA.
Sec. BB.
Section CC.
Section DD.
Section EE.
Section FF.
The lines XX are in one plane at right angles to axis of rotation.

FIG. 55.

(*b*) *The S.E.* 5 *Aeroplane**

A rather comprehensive series of experiments was made at the National Physical Laboratory with models of seven airscrews,† each designed for the 150 h.p. ungeared Hispano-Suiza engine, and a model of the S.E. 5 aeroplane. The experiments are of special interest as showing how the resistance of an aeroplane at the same forward speed and thrust depends on the characteristics of the airscrew. It is proposed not to describe in detail the performance of these seven airscrews, which were of widely different characteristics. It is thought sufficient to give only the performance of the standard S.E. 5 airscrew—T.28066 for the 150 h.p. Hispano-Suiza engine—and to present only the final conclusions of the investigation. A sketch of this standard two-bladed airscrew, which

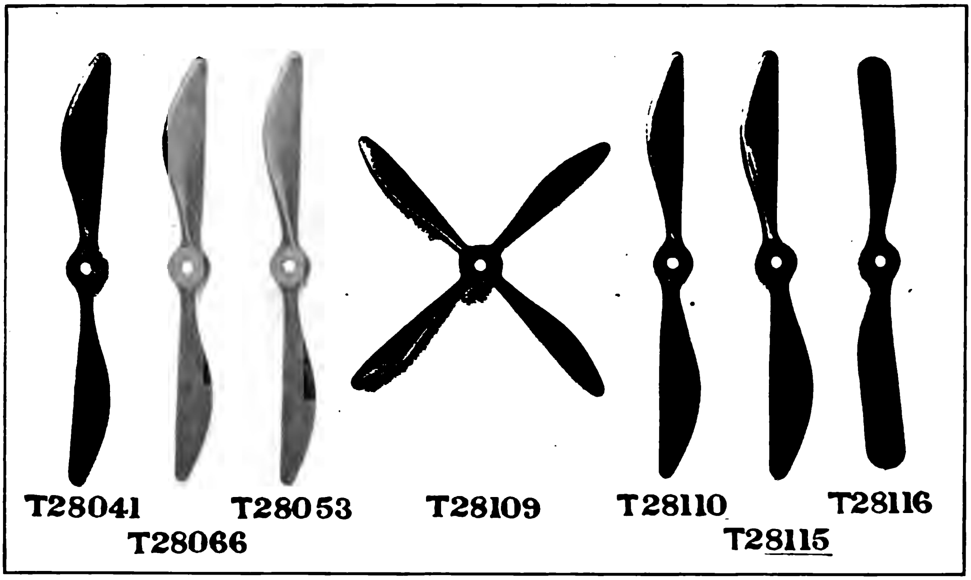


FIG. 56.

was the most efficient of the seven airscrews, is shown in Fig. 55. The diameter of the airscrew is 7 ft. 10 in. A photograph of the seven airscrews is given in Fig. 56.

A sketch of the model of the S.E. 5 machine is shown in Fig. 57, from which it will be seen that the body, wings, wing struts, tail plane, landing gear, and gun are represented. The radiators at the front of the aeroplane were represented in the model by wire gauze of such a mesh that its resistance coefficient was approximately equal to that of the radiator.

The performance of the standard airscrew with the interference of the aero-

* "Some experiments with a model of the S.E. 5 aeroplane and models of several airscrews designed for the 150 h.p. ungeared Hispano-Suiza engine," by A. Fage, A.R.C.SC., G. A. Hankins, A.R.C.SC., and R. G. Howard, B.SC. Advis. Comm. Aeron., 1919.

† The performances of the full-scale airscrews were also measured in flight at the Royal Aircraft Establishment.

plane is shown graphically in Fig. 58. It will be seen from this figure that the airscrew has a maximum efficiency of 82 per cent.

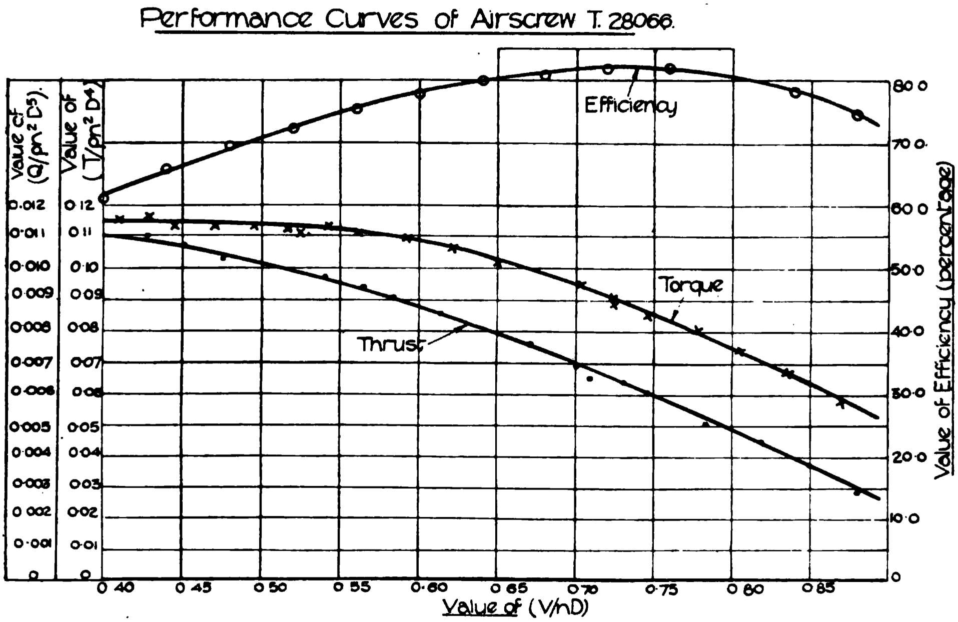


FIG. 58.

A Comparison of the Performances of Seven Airscrews Designed for the S.E. 5 Aeroplane

From a comparison of the performances of the seven airscrews we found :—

(*a*) That the performance of an airscrew with the rectangular plan form and semicircular blade tips, namely T.28116, compared very favourably with that of the standard airscrew. On an average this airscrew, when mounted on the same engine, develops at the same forward speed about 2·5 per cent less thrust than the standard airscrew.

(*b*) The performance of a four-bladed airscrew, T.28109, was distinctly inferior to that of the standard. This was probably because of the lower diameter, 6 ft. 6 in. as compared with 7 ft. 10 in. We calculated from the performance data that at the same forward speed the four-blader runs about 2 per cent faster and develops about 5 per cent less thrust than the two-blader.

(*c*) From a comparison of the performances of two airscrews, namely T.28066 and T.28110, which differ only in the type of blade section—with airscrew T.28110 the curved surface of each blade section was a circular arc, with the maximum thickness midway between the leading and trailing edges, whereas each blade section of T.28066 was of the ordinary aerofoil type, with the maximum ordinate at about one-third of the chord from the leading edge—we found that the aerofoil type of blade section was more efficient aerodynamically than the circular arc type. At any value of (V/nD) the efficiency of T.28110 was about

3 per cent lower than that of airscrew T.28066. When mounted on the standard engine airscrew T.28110 would run about 2·5 per cent faster and would develop about 2·2 per cent less thrust over the working range of forward speed.

(*d*) Some experiments with a "climbing" airscrew, T.28115, showed fairly conclusively that an airscrew to be efficient at the climb must sacrifice efficiency at the maximum horizontal flight speed. As would be expected, this airscrew had its maximum efficiency at a low value of the forward speed. Also the rotational speed was fairly high at a high forward speed, so that unless the throttle were suitably adjusted the rotational speed would become excessive at the maximum flying speed of the aeroplane. At climbing, this airscrew developed a greater thrust than any of the other airscrews of the series, chiefly because of the greater horse-power absorbed. Thus at a forward speed of 60 m.p.h. the airscrew gave about 5·5 per cent more thrust than the standard airscrew, but to do so absorbed about 5·5 per cent more horse-power. The efficiencies of these two airscrews at this forward speed were almost equal. Compared with the other airscrews the thrust of T.28115 diminished very rapidly with an increase of forward speed, especially when the engine was throttled down to a safe maximum speed of rotation.

Dependence of the Resistance of an Aeroplane on the Characteristics of the Airscrew

The most interesting feature of these experiments was that the interference on the resistance of the aeroplane of each airscrew—designed for the same engine and aeroplane—was shown to depend on the characteristics of the airscrew. The resistances of the complete machine without those parts of the wings which have a dihedral angle were measured (*a*) without the airscrew in position and (*b*) with the airscrew rotating at conditions similar to those of practice. Each resistance measurement includes, however, the interference of the outer parts of the wings. For each combination of airscrew and aeroplane the relationship between (R/R_0) and $(T/\rho V^2D^2)$ may be expressed by the linear equation $(R/R_0)=K_1+K_2(T/\rho V^2D^2)$. The values of K_1 and K_2 are given in Table VII.

TABLE VII

Airscrew.	Diameter, feet.	Number of blades.	Value of K_1	Value of K_2
T.28041	8·0	2	0·833	1·04
T.28066	7·83	2	0·860	1·11
T.28053	8·0	2	0·832	1·01
T.28110	7·83	2	0·868	1·01
T.28115	7·83	2	0·892	1·03
T.28116	7·5	2	0·827	1·03
T.28109	6·5	4	0·920	1·05

The value of R_0 for the full-size aeroplane—without the wings, which have a dihedral angle—in a wind of 100 ft. per sec. at ground level was calculated from the model experiments to be 112 lb.

From the data of the above table it will be seen that at constant values of the thrust and the forward speed the resistance of an aeroplane depends appreciably on the type of airscrew mounted. Thus a change from the two-bladed airscrew T.28053 to the two-bladed airscrew T.28115—the diameters of these airscrews are almost equal—increases the resistance of the aeroplane at forward speeds of 60 m.p.h. and 110 m.p.h. by about 2·5 per cent and 5 per cent respectively of the thrust. With the four-bladed airscrew instead of the two-bladed airscrew T.28053—diameters 6 ft. 6 in. and 8 ft. 0 in.—the increases of the resistance of the aeroplane expressed as a fraction of the thrust are 5·5 per cent and 10 per cent at forward speeds of 60 m.p.h. and 110 m.p.h. respectively. Appreciable error may result, therefore, in the calculation of the resistance of an aeroplane if it be assumed that the augmented resistance of the aeroplane due to the working of the airscrew depends, for airscrews of the same diameter, only on the thrust and the forward speed. Such were the assumptions of the hypothetical formula on p. 81.

The Mutual Interference of an Airscrew and a Tractor Body, as Affected by the Fairing of the Nose of the Body

An investigation* of a very general character was made by the author to determine the effect of fairing the nose of a tractor body, the fairing being made either behind the airscrew or in front by a faired nose-piece attached to, and rotating with, the airscrew.

The experiments were made with a model body of B.E. 2c type, modified at the nose so as to facilitate the attachment of a fairing piece, whilst at the same time retaining a shape characteristic of an aeroplane body. The need for, and the nature of, the fairing at the front of the body depends greatly on the type of engine mounted. Thus, in the case of a stationary air-cooled engine it may be of greater importance to make the air flow around the engine and so obtain more efficient cooling, rather than to attempt to reduce the resistance by making the flow conformable to the lines of the body. With an engine driven by a water-cooled engine much depends on the position of the radiator. Obviously, fairing would not be desirable if the radiator were mounted at the nose of the body, but unfortunately the flow of air through the radiator would probably greatly increase the drag of the body.

As stated previously, these experiments were essentially of a preliminary character, and accordingly it is considered not necessary to give any experimental data but to discuss in outline the general conclusions of the investigation. A sketch of the several combinations of airscrew and body, with which the experiments were made, is shown in Fig. 59. The forward end of the body was so

* "A preliminary investigation of the mutual interference of an airscrew and a tractor body, as affected by the fairing of the nose of the body," by A. Fage, A.R.C.SC., and H. E. Collins. Advis. Comm. Aeron., 1918.

shaped as to receive a faired nose-piece of circular transverse section. This nose-piece was detachable. The body with the faired nose-piece attached is known as the "faired" body; with the nose-piece detached as the "unfaired" body. The nose-piece was also adapted to fit round and to rotate with the airscrew, as shown in combination 5 of Fig. 59. The mutual interferences of combinations 2, 3, and 5, that is the performances of both airscrew and body, with and without mutual interference, were measured. Summarising the interesting features of this investigation we found :—

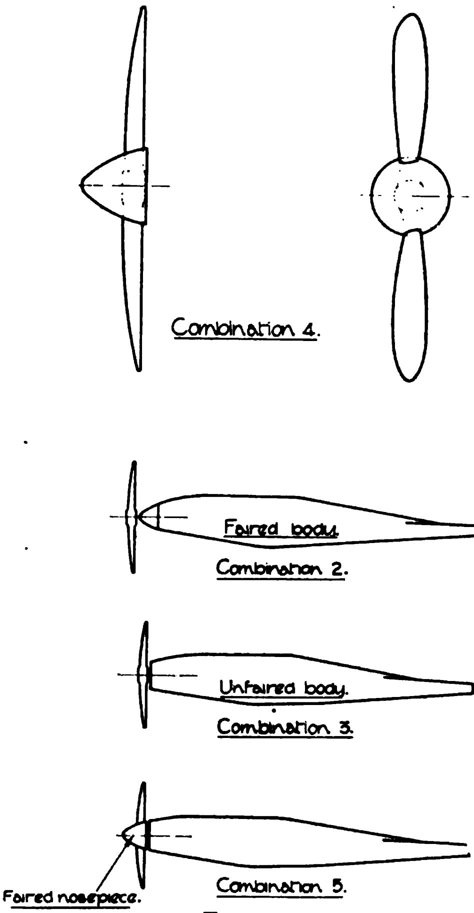


FIG. 59.

(*a*) That the performance of the airscrew alone was unaffected within the limits of experimental accuracy by the attachment of the faired nose-piece. This result is probably only a characteristic of this particular combination of airscrew and faired boss, and is not of general applicability.

(*b*) The presence of each body increased at the same value of (V/nD) both the thrust and the torque of the airscrew. The presence of the faired body—combination 2—did not appreciably modify the performance of the airscrew. The effect of the body was more marked with combination 3. The maximum interference effect, and also the maximum increase of efficiency was, however, obtained with combination 5, that is the "unfaired" body and airscrew with faired nose-piece. This advantage was mainly brought about by the fact that whereas at the given values of V and n the thrust was increased by about the same percentage as in combination 3, the increase of torque was much smaller.

(*c*) As with other resistance experiments, we found that the relationships between the values of (R/R_0) for each body of the three combinations, and the corresponding values of $(T/\rho V^2D^2)$ for the airscrew as affected by the resistance of the body, could be expressed by linear equations, the constants of each linear equation depending on the characteristics of each combination. To determine whether there is any advantage to be gained by fairing the nose of the body, the fairing being made either behind the body or the shape of the body being continued in front by a nose-piece attached to and rotating with the airscrew, the performances of the several combinations of airscrew, engine, and body were compared. We found that there was no advantage in the use of the "faired" body as in combination 2 as compared with the "unfaired" body as in com-

bination 3. With combination 5 as compared with combination 3 there would appear to be an appreciable gain of performance over the whole of the working range of (V/nD). It would seem, then, that with the particular bodies and airscrew of these experiments, the best combination was that in which the airscrew, having attached to it the faired nose-piece, rotated in front of the "unfaired" body. Also that no appreciable advantage was to be obtained by fairing the nose of the body behind the airscrew.

It is perhaps unwise to generalise from the results of such a particular series of experiments, but it would seem that wherever possible fairing the nose of a body by a nose-piece attached to and rotating with the airscrew is to be recommended.

The Mutual Interference of an Airscrew, a Rotary Engine, and the Body of a Tractor Aeroplane*

We have up to the present considered the mutual interference of an airscrew and an aeroplane with a stationary engine. It is now proposed to consider in

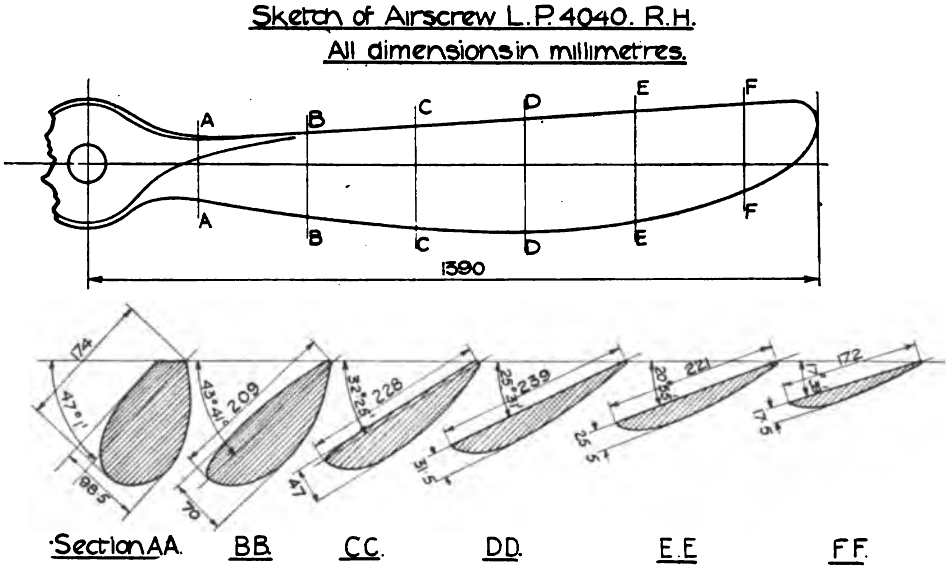


FIG. 60.

some detail the mutual interference of an airscrew and an aeroplane with a rotary engine. A rather comprehensive series of experiments with models was made at the National Physical Laboratory to measure the overall performance of the combination of airscrew, rotary engine, and body of the Sopwith Snipe aeroplane, and also to obtain for flight conditions similar to those of practice the individual performances of the airscrew, engine, and body. The question as

* "Some experiments with models of an airscrew, a rotary B.R. 2 engine and two different types of engine cowling, and a body of the Snipe aeroplane," by A. Fage, A.R.C.SC., and H. E. Collins. Advis. Comm. Aeron., 1919.

to whether it is advisable, in so far as aerodynamic considerations are concerned, to fair the nose of the body by a spinner attached to and rotating with the airscrew was also investigated.

A sketch of the two-bladed airscrew L.P.4040, of which a model of one-third scale was used in the experiments, is shown in Fig. 60. This airscrew has two

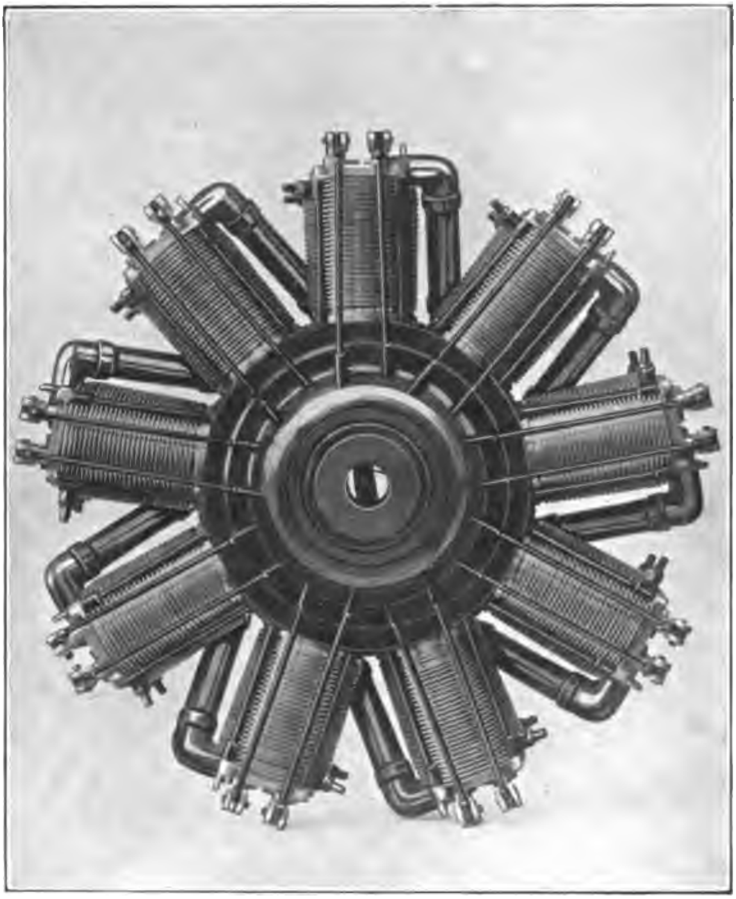


Fig. 61.

blades and a diameter of 9 ft. 1·5 in. A photograph of the model of the B.R. 2 engine is given in Fig. 61. This engine has nine cylinders, each of bore 140 mm. and of stroke 180 mm., an overall diameter of 42·5 in., and develops about 230 h.p. at 1260 r.p.m.

Experiments were made with two engine cowlings. The shape of model cowling A is given in Fig. 62. The lower part of this cowling is cut away, the part of the body immediately behind being scooped out in such a manner as to make a ready passage of escape for the air within the cowling. It will be seen

from Fig. 63 that cowling B is made in two parts :—(*a*) A cylindrical part within which the engine rotates and which is attached to the body. The bottom of this part of the cowling is not cut away. (*b*) A faired nose-piece or spinner which is

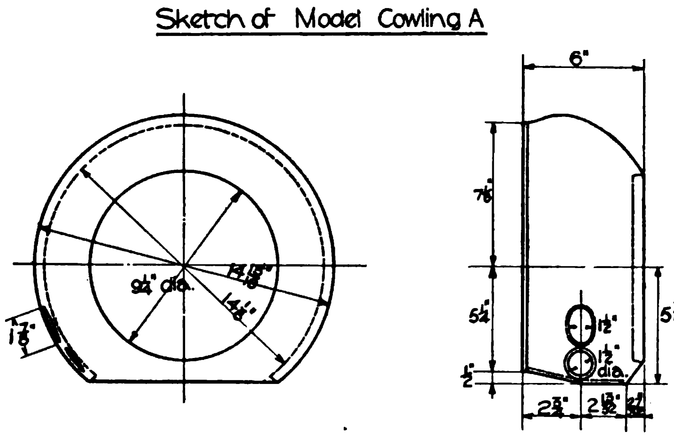


FIG. 62.

attached to and rotates with the airscrew. There is a circular opening at the nose of the spinner.

With this cowling, therefore, air is admitted to the engine through the circular opening at the front of the nose-piece and the annular opening between the rotating and fixed parts of the cowling, and is then discharged through the " scoop " at the bottom of, and the triangular openings at, the sides of the body.

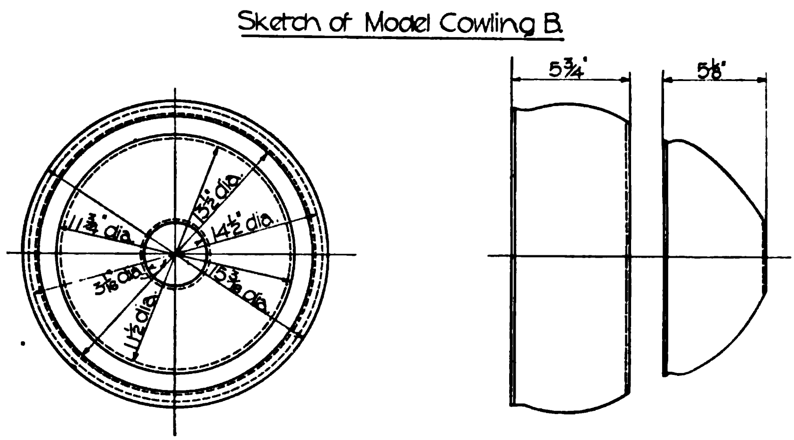


FIG. 63.

A sketch of the model of the " Snipe " body is given in Fig. 64. It will be seen that the nose of the body, on which is fitted the cowling, is circular in front elevation. The bottom of the front part of the body is scooped out and triangular

air-ducts are cut at the sides. The two machine-guns at the top of the body and in front of the pilot's seat were represented on the model. The model body was not fitted with wings, landing gear, tail plane, etc.

The data of Figs. 65–67 show for cowling A and also for cowling B—

(1) The performance of the airscrew alone with the interference of the combination of the body and the engine rotating in the cowling.

(2) The performance of the combination of the airscrew and engine, with the interference of the body with the cowling.

(3) The performance of the combination of the airscrew, engine rotating in the cowling, and body.

With the combinations with cowling B, the spinner is regarded as part of

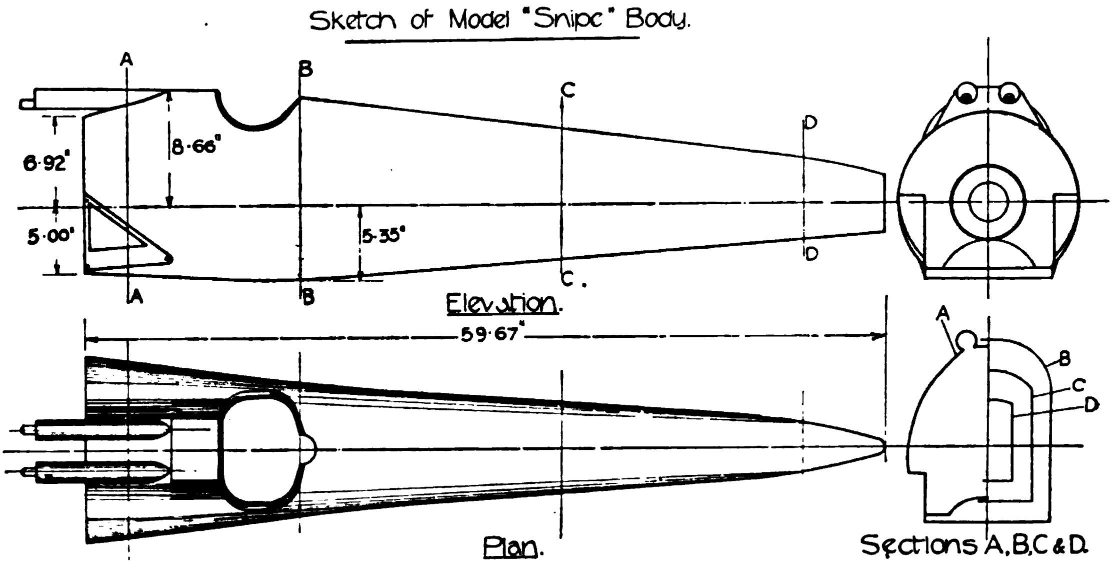


FIG. 64.

the airscrew. With the interference of the engine rotating in cowling A, it is seen from Fig. 67 that the airscrew has a maximum efficiency of 83·5 per cent. The change of cowling and the attachment of the spinner to the airscrew reduces the maximum efficiency to 80·5 per cent, the drop of efficiency at the same value of (V/nD) being largely due to fall of thrust. It is impossible to say from the experimental data whether the attachment of the spinner to the airscrew or the change of the shape of the fixed portion of the cowling has the greater influence on the airscrew performance. The experimental data show that the performance of the combination of airscrew and engine is greatly dependent on the type of engine cowling mounted. Thus a change of cowling from " A " to " B " increases the maximum efficiency of the combination from 62·0 per cent to 68·5 per cent, the engine in the latter case having both a smaller windage loss and a smaller resistance. It should be noted that a sound comparison of the merits of the

Data of Experiments made with Models of Airscrew L.P. 4040 and B.R. 2 Engine, mounted on Cowling "A" on the "Snipe" Body.

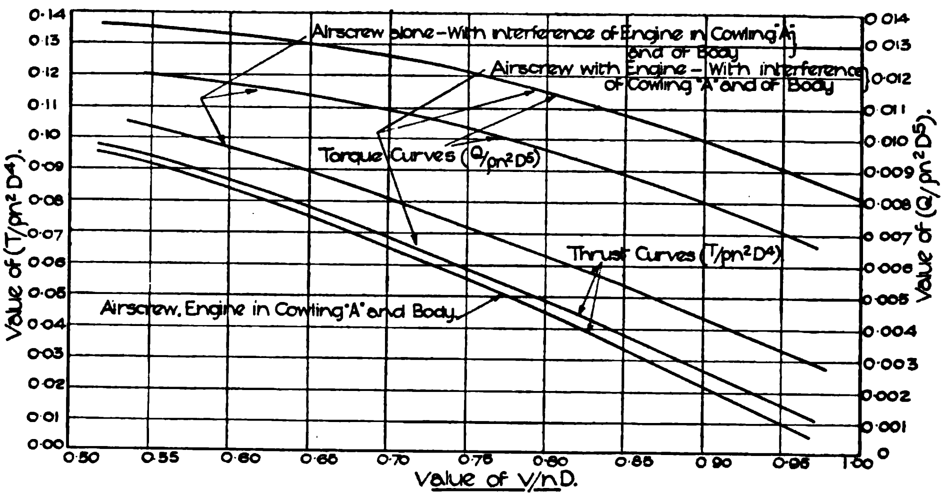


FIG. 65.

Data of Experiments made with Models of Airscrew L.P. 4040 & B.R. 2 Engine, mounted in Cowling "B" on the "Snipe" Body.

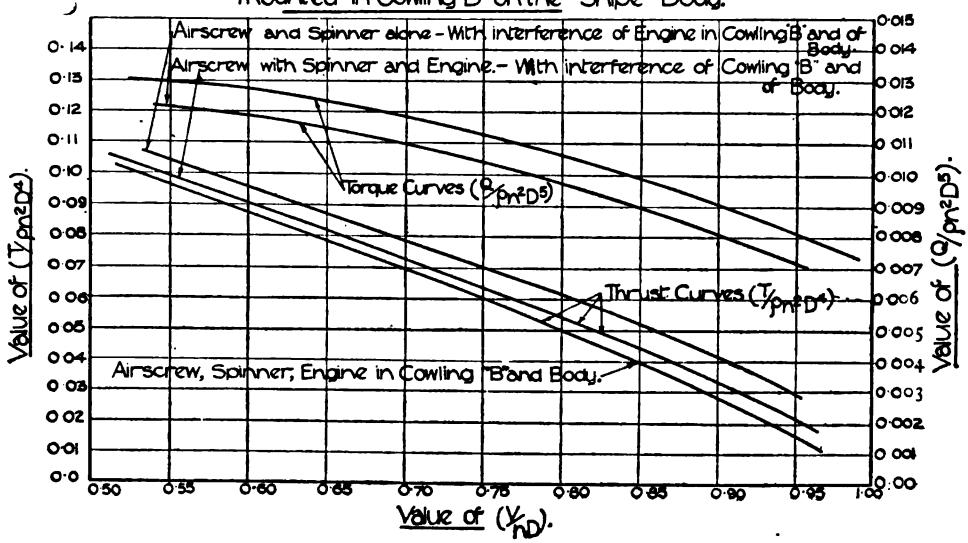


FIG. 66

two types of engine cowling can only be made when it is known how the working of the engine is affected by the quantity of air flowing through the cowling. It may be that the engine is overcooled in cowling A, or not sufficiently cooled in cowling B. From the experimental data of Fig. 67 there is no doubt, however, that aerodynamically, cowling B is much better than cowling A.

The investigation shows in a very pronounced manner that the design of an engine cowling is of great importance, since the performance of an aeroplane with a rotary engine will depend not only on the type of cowling, but also on whether the correct quantity of air needed for efficient cooling is admitted to the engine.

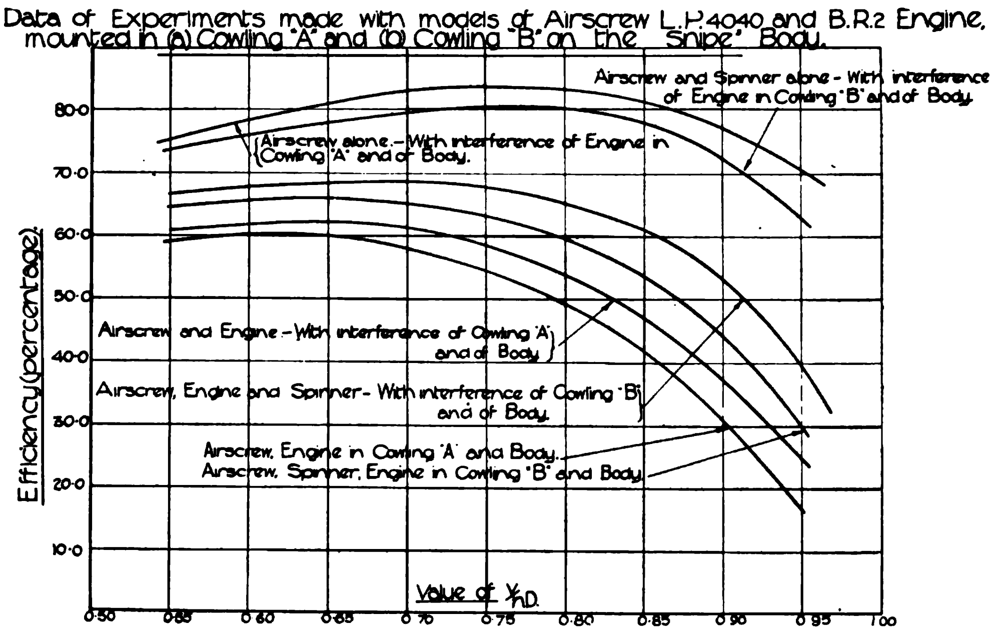


FIG. 67.

The curves showing the variation of the thrust of the combination of airscrew, engine, body, and cowling with the (V/nD) of the airscrew are given in Figs. 65 and 66. From the performance curves of Fig. 67 it will be seen that the maximum efficiency of the combination with cowling A is 60 per cent, and with cowling B, 66 per cent. The efficiency in each case is the ratio of the useful work done by the thrust of the complete combination to the work of the torque of the combination of airscrew and engine.

The data of the aerodynamic performance of the full-size engine when working in either cowling A or cowling B are shown graphically in Fig. 68. With these curves Q_E represents the torque in lb.-ft. and R_E the resistance in lb. It will be noticed that at the same forward speed the windage torque of the engine with the airscrew rotating is smaller than with the airscrew not rotating. At any value of (V/nD) the windage torque, with the airscrew rotating at the same speed

as the engine, is 87 per cent with cowling A, and 77 per cent with cowling B, of the value without the airscrew rotating. At a constant value of the rotational speed the windage torque increases with an increase of the forward speed, both when the airscrew is not rotating and when rotating at the same speed as the engine.

An analysis of the horse-power account of the airscrew and engine—with either cowling A or cowling B—at two conditions of flight at ground level, namely, (*a*) V=80 m.p.h., N=1225 r.p.m., and (*b*) V=110 m.p.h., N=1330 r.p.m., is given in Table VIII. From the data of this table it will be seen that the windage

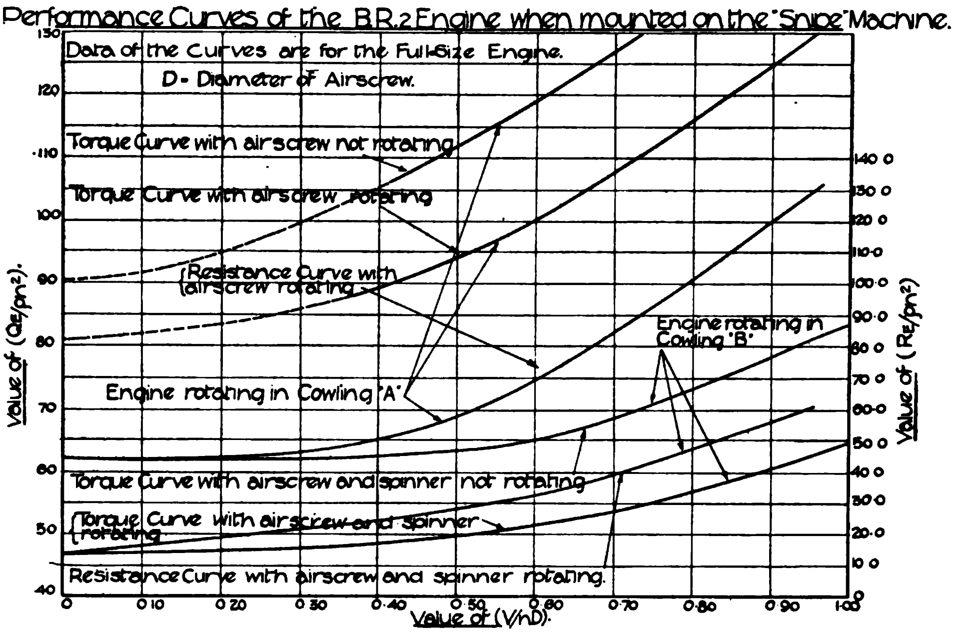


FIG. 68.

horse-power of a rotary engine is an important factor in so far as the aerodynamic performance of an aeroplane is concerned. Thus taking an average value over the working range of the flight speed, the windage horse-power with the engine rotating in cowling A is about 14 per cent of the horse-power absorbed by the airscrew and engine—exclusive of mechanical losses—and about 20 per cent of the thrust horse-power. The corresponding values of these ratios with the engine working in cowling B are 7·5 per cent and 10 per cent respectively. It will also be seen that calculations made from windage experiments on the engine with no wind blast and with the airscrew not rotating underestimate the windage horse-power of the engine in flight.

Curves showing the variation of the resistance of the engine with (V/nD) of the airscrew are given in Fig. 68. At any rotational speed the resistance of the engine increases rather rapidly with an increase of the forward speed. At a

TABLE VIII

Analysis of the Horse-Power Account at Various Conditions of Flight

Calculations made for density of air at ground level.

	Conditions of flight. V = 0. Windage h.p. when		Conditions of flight. V = 80 m.p.h. N = 1225 r.p.m.				Conditions of flight. V = 110 m.p.h. N = 1330 r.p.m.			
	N = 1225 r.p.m.	N = 1330 r.p.m.	Thrust h.p.	H.p. absorbed by airscrew.	Windage h.p.	*H.p. absorbed by air-screw and engine.	Thrust h.p.	H.p. absorbed by airscrew.	Windage h.p.	*H.p. absorbed by air-screw and engine.
Engine rotating in cowling A. Airscrew rotating at same speed in front.	18·7	23·8	135·5	169·0	23·0	192·0	153·0	183·5	34·0	217·5
Engine rotating in cowling A. Airscrew not rotating.	20·9	26·6	—	—	28·1	—	—	—	38·1	—
Engine rotating in cowling B. Airscrew and spinner rotating at same speed in front.	10·9	13·8	130·0	169·0	12·5	181·5	147·5	183·5	16·5	200·0
Engine rotating in cowling B. Airscrew not rotating.	14·3	18·2	—	—	15·3	—	—	—	21·5	—

* The values of these columns do not include mechanical losses of engine.

forward speed of 110 m.p.h. and a rotational speed of 1330 r.p.m. the resistances of the engine, (*a*) in cowling A, and (*b*) in cowling B, are 22·5 per cent and 11 per cent respectively of the airscrew thrust ; at the climbing speed of 80 m.p.h. and the rotational speed of 1225 r.p.m. the values are 11·5 per cent and 8·0 per cent respectively.

The two most important conclusions which may be made from the data of these experiments are, firstly, the design of an engine cowling is of great importance from both the aerodynamic and engine-cooling standpoints ; and, secondly, it would appear advantageous aerodynamically to fair, by means of a spinner attached to and rotating with the airscrew, the body of a tractor aeroplane driven by a rotary engine.

THE WINDAGE LOSSES OF A ROTARY ENGINE

From the experiments just described, we see that the windage losses appreciably influence the performance of an aeroplane.

Although perhaps a little outside the scope of the present work, it is thought desirable to consider still further this subject. A few months before making the experiments with the model of the B.R. 2 engine, some experiments* were made to measure the windage losses on a model of the B.R. 1 engine, when rotating in front of a model of the body of the Sopwith " Camel " aeroplane, at conditions similar to those of practice.

The B.R. 1 engine has nine cylinders, each of bore 120 mm., an overall diameter of 42 in., and develops 150 h.p. at a rotational speed of 1250 r.p.m. It is proposed not to give a detached analysis of the data of these experiments, but only a summary of the general conclusions. We found—

(*a*) That the cowling reduces the windage torque of the engine. Making comparisons at constant values of the forward and rotational speeds, the windage torque with the cowling varies from 65 per cent to 70 per cent of the value without the cowling, both with and without the airscrew rotating.

(*b*) At any forward speed there is a blocking-up effect due to the rotating airscrew, that is the windage torque of the engine with the airscrew rotating at the same speed is lower than when the engine rotates alone.

(*c*) At any rotational speed the windage losses increase with an increase of the forward speed of the aeroplane. We found with this particular engine and cowling, that the windage losses at the maximum horizontal flight speed and at climbing are greater by about 37·5 per cent and 12 per cent respectively, than the values calculated from experiments where the engine rotates at a stationary point without a wind blast.

(*d*) As would be expected, the interference of the wings on the windage torque of the engine is practically negligible.

(*e*) Keeping the values of the forward speed of the aeroplane and the rotational speed of the engine and airscrew constant, there appears to be a definite area of the circular hole at the front of the cowling, below which the windage losses are

* " Windage experiments with a model of the rotary engine B.R. 1.," by A. Fage, A.R.C.SC., and H. E. Collins. Advis. Comm. Aeron., 1918.

dependent on the area of the circular opening, but above which the losses remain fairly constant, and depend on the " drawing-in " capacity of the engine. It will, of course, be obvious that by suitable variation of this circular opening the windage losses can be maintained constant at all speeds of flight.

(*f*) When the forward speed is zero, closing the openings at the circumference and the back of the cowling so that the only opening is the circular hole at the front reduces the windage losses by about 50 per cent.

CHAPTER VIII

TANDEM AIRSCREWS

THE large aeroplane has of necessity to be a multi-engined machine because the maximum horse-power which can be absorbed with good efficiency by a single airscrew is limited, *inter alia,* by the diameter and also by the maximum stress permissible in the material of the airscrew blade. With such an aeroplane the arrangement of the power units needs careful consideration. The tandem combination of airscrews favours compactness and lightness of construction. The question which is of some concern to an aeroplane designer is whether such advantages compensate for the loss of efficiency of the rear airscrew when working in the outflowing stream from the forward airscrew. It is proposed, then, to consider how the performance of an airscrew is modified when working as the forward or rear airscrew of a tandem combination. At the outset we shall consider the general theory of tandem airscrews.* Afterwards the subject will be considered from the experimental standpoint.

APPLICATION OF THE AEROFOIL THEORY TO THE CASE OF AN AIRSCREW WORKING IN THE OUTFLOWING STREAM OF ANOTHER

It has already been shown that in the case of a single airscrew the thrust on a blade element is

$$\frac{\rho.K_L.C.dr.(V+aV)^2}{\tan\psi.\sin\psi}\left[1-\frac{K_D}{K_L}\tan\psi\right];$$

also that the efficiency of working is $\dfrac{1}{(1+a)}\,\dfrac{\tan\psi}{\tan(\psi+\gamma)}$,

where $\tan\psi=\dfrac{(1+a)V}{2\pi rn}$.

If aV be the inflow velocity into a blade element, then the velocity of the outflowing stream has been represented by abV, where the value of " b " may in the present investigation be taken as 2.

We shall now consider the case of a blade element of the rear airscrew which is working in the outflowing stream of the forward airscrew, the magnitude of this velocity measured relative to undisturbed air being represented by abV. We have to find, then, the performance of a blade element of the rear airscrew which is moving in a stream of general velocity $(1+ab)V$, where V is the velocity of the airscrew relative to the air undisturbed by the tandem

* The effect of the inflowing velocity of the air on the efficiency of an airscrew, with a special reference to the case of tandem airscrews of large machines. By A. Fage, A.R.C.SC., D.I.C.

combination. Proceeding as in the case of a single airscrew, it follows that the elemental thrust

$$=\frac{\rho.C.dr.K_L(V+abV)^2(1+a_1)^2}{\tan\psi.\sin\psi}\left[1-\frac{K_D}{K_L}\tan\psi\right].$$

Also the elemental torque

$$=\frac{\rho.C.dr.K_L.r.(V+abV)^2(1+a_1)^2}{\tan\psi.\sin\psi}\left[\frac{K_D}{K_L}+\tan\psi\right],$$

where $a_1(1+ab)V\equiv$inflow velocity of air at the blade element under consideration,

and $\tan\psi=\dfrac{(1+ab)\,(1+a_1)V}{2\pi rn}$.

Also the efficiency, $\eta=\dfrac{V\left(1-\frac{K_D}{K_L}\tan\psi\right)}{2\pi.n.r\left(\frac{K_D}{K_L}+\tan\psi\right)}$

$$=\frac{1}{(1+ab)\,(1+a_1)}\,\frac{\tan\psi}{\tan(\psi+\gamma)}.$$

The thrust and torque of the rear airscrew may then be obtained directly by the integration of the thrust and the torque of each element of the blade. The assumptions here made, in addition to those of the ordinary aerofoil theory, are that the rear airscrew works completely within, and does not modify, the outflowing stream from the forward airscrew, and also that the radial position of the element of a rear blade is known at which the velocity is $(1+ab)V$.

Application of the Froude Theory to the Case of an Airscrew Working in the Outflowing Stream of Another

The fundamental assumptions of the Froude theory are applied to the present problem. In addition it is assumed that the rear airscrew works completely in the outflowing stream, of magnitude $2aV$, of the forward airscrew.

Proceeding in the manner suggested by the Froude theory, it follows that the velocity of the inflowing air due to the thrust of the rear airscrew $=a_1(1+2a)V$.

Also the mass of air flowing through the airscrew $=\dfrac{\pi D^2}{4}\rho(1+2a)\,(1+a_1)V$.

Assuming the velocity of the air in the outflowing stream of the rear airscrew to be $(2aV+V_0)$, this velocity is taken relative to the undisturbed air, and accepting Froude's hypothesis, the thrust

$$=\frac{\pi D^2V}{4}\rho(1+2a)\,(1+a_1)V_0$$

$$=\frac{\pi D^2}{4}\,\frac{\rho}{2}\,[(2aV+V+V_0)^2-(2aV+V)^2.]$$

Hence $\quad \dfrac{V_0^2}{2}+V_0(2a+1)V=V(1+2a)\,(1+a_1)V_0,$

that is $\quad V_0=2a_1(1+2a)V.$

The velocity of the outflowing stream then becomes $(1+2a)V.2a_1$. The increase of the kinetic energy of translation of air passing through the airscrew disc, due to the work done by the thrust of the airscrew,

$$=\frac{1}{2}\left(\frac{\pi D^2}{4}\right)\rho(1+2a)\,(1+a_1)V\,[(2aV+V_0)^2-(2aV)^2]$$

$$=\frac{\pi D^2}{8}\rho(1+2a)\,(1+a_1)V.\,(4aV+V_0)V_0=\frac{T}{2}(4aV+V_0).$$

Energy absorbed by the airscrew in unit time = Useful work done in unit time + Increase of K.E. of translation of the air passing through the airscrew

$$=VT+T\left(2aV+\frac{V_0}{2}\right)=TV(1+a_1)\,(1+2a).$$

From the preceding expression it will be seen that the total work done by the airscrew in unit time is equal to the product of the thrust and the velocity of the air passing through the airscrew disc.

$$\text{The efficiency of working }=\frac{VT}{VT(1+a_1)\,(1+2a)}$$

$$=\frac{1}{(1+a_1)\,(1+2a)}.$$

The Probable Efficiency of the Rear Airscrew of a Tandem Combination

We see from the two preceding expressions for the efficiency of a rear airscrew of a tandem combination that, as in the case of a single airscrew, the inflow factor of the efficiency expression, namely $\frac{1}{(1+a_1)\,(1+2a)}$, as calculated from the aerofoil theory, has the same value as the efficiency as calculated directly from the Froude theory. This factor has, however, a smaller value when an airscrew is working at the rear of a tandem combination. For airscrews of the same diameter giving the same thrust at the same forward speed of the aeroplane, the value of the factor $\left(\frac{\tan\psi}{\tan\,(\psi+\gamma)}\right)$ may be regarded with good accuracy to have the same value for either a forward or a rear airscrew. Assuming this to be the case, it follows that the efficiency of the rear airscrew is $\frac{(1+a)}{(1+2a)\,(1+a_1)}$ of the efficiency of the forward airscrew.

It is of some interest to obtain for the working conditions of practice approximate theoretical values of this ratio. To do so we shall assume firstly that the diameters of the forward and rear airscrews are equal, and secondly that the airscrews are giving equal thrusts at the same forward speed of the aeroplane. Adopting the "Froude" expression for thrust and assuming that the rear airscrew is immersed completely in the outflowing stream of the forward airscrew, it then follows that

$$a(1+a)=a_1(1+a_1)\,(1+2a)^2.$$

It will, of course, be realised that the velocity and the area of the outflowing

stream of the front airscrew, in so far as the rear airscrew is concerned, will depend, *inter alia*, on the distance between the airscrews.

Regarding the average values of " a " for a modern airscrew to be 0·04 at the maximum horizontal flight speed and 0·10 at the climb, the corresponding values of " a_1 " would be 0·035 and 0·075 approximately.

Substituting these values of " a " and of " a_1 " in the expression

$$\frac{(1+a)}{(1+2a)\,(1+a_1)}$$

it follows, theoretically, that the efficiency of a rear airscrew is 0·93 per cent at the maximum horizontal flight speed, and 0·85 per cent at the climb, of the efficiency of a forward airscrew designed for an engine of equal horse-power. It will be seen later that these values are in very close agreement with those calculated from experimental data.

Experiments with Tandem Combinations of Airscrews

The preceding discussion shows in a general theoretical manner, how the performance of an airscrew is affected when working at the rear of a tandem

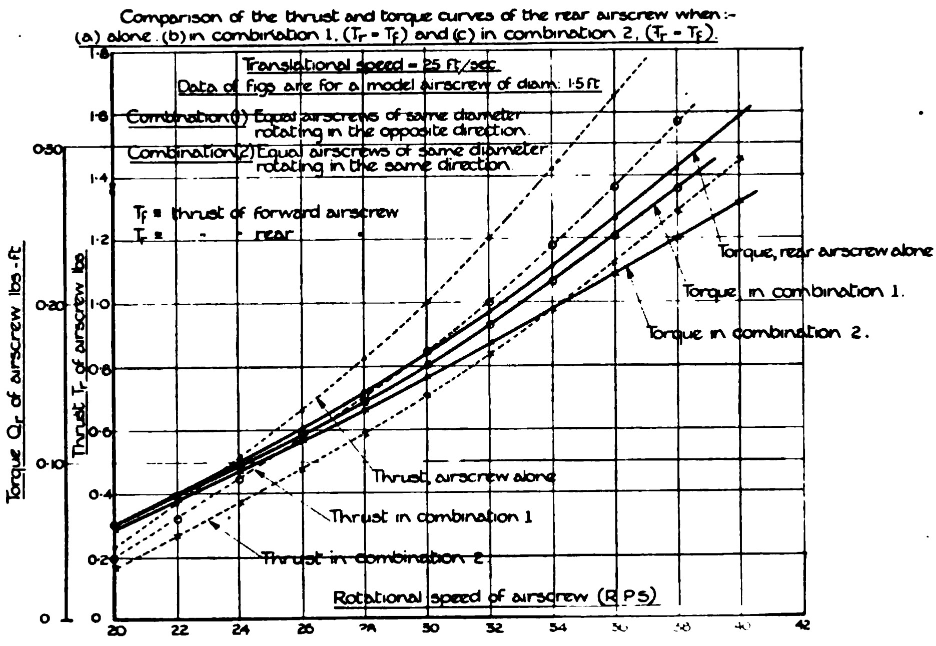


FIG. 69.

combination. Before the general conclusions of such an investigation can be accepted it is necessary to substantiate them by experiment. Some preliminary experiments* of a somewhat general character have been made at the National

* " Some experiments with combinations of tandem airscrews," by A. Fage, A.R.C.SC., and H. E. Collins. Advis. Comm. Aeron., 1918.

Attention should here be directed to a comprehensive series of experiments on tandem airscrews made by G. Eiffel and described in *L'Aerophile*, Nov., 1919. The present work had gone to the Press before the publication of this article.

Physical Laboratory, to show how the performance of an airscrew is modified when working in the outflowing stream of a forward airscrew ; a comparison was made of the performance of the airscrew *alone*, with the performance of the same airscrew when mounted at the rear of a tandem combination. In some respects it would have been better, perhaps, to have made a comparison between the performance of an airscrew designed to develop a given thrust at a given rotational speed and *another* airscrew, of the same plan form and shape of blade sections, *designed* to develop, when working in the outflowing stream of a forward airscrew, an equal thrust at the same value of the rotational speed, the forward speed relative to undisturbed air being the same in each case. In the latter case, the blade angles of the rear airscrew would be increased to take account of the additional translational and rotational velocities of the outflowing stream from

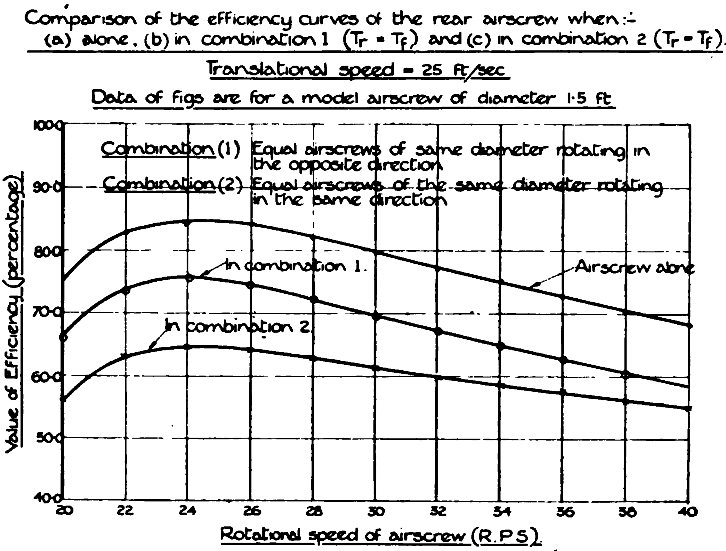


FIG. 70.

the forward airscrew. The general results of such an investigation would be of the same character as those of the present, but it is thought desirable to point to the *slight* difference between experiments on tandem combinations—where the rear airscrew is especially designed for such a combination—and experiments made to measure how the performance of any airscrew is affected by the outflowing stream from another.

Experiments were made with four tandem combinations in which (*a*) airscrews of different diameter rotated in the same direction, and (*b*) airscrews of the same diameter rotated (1) in the same direction, and (2) in the opposite direction. With the latter series of experiments, the interferences on the same rear airscrew of two equal forward airscrews, but of opposite direction of rotation, were found. The performances were measured of the rear airscrew when working alone, and when developing the same thrust as the

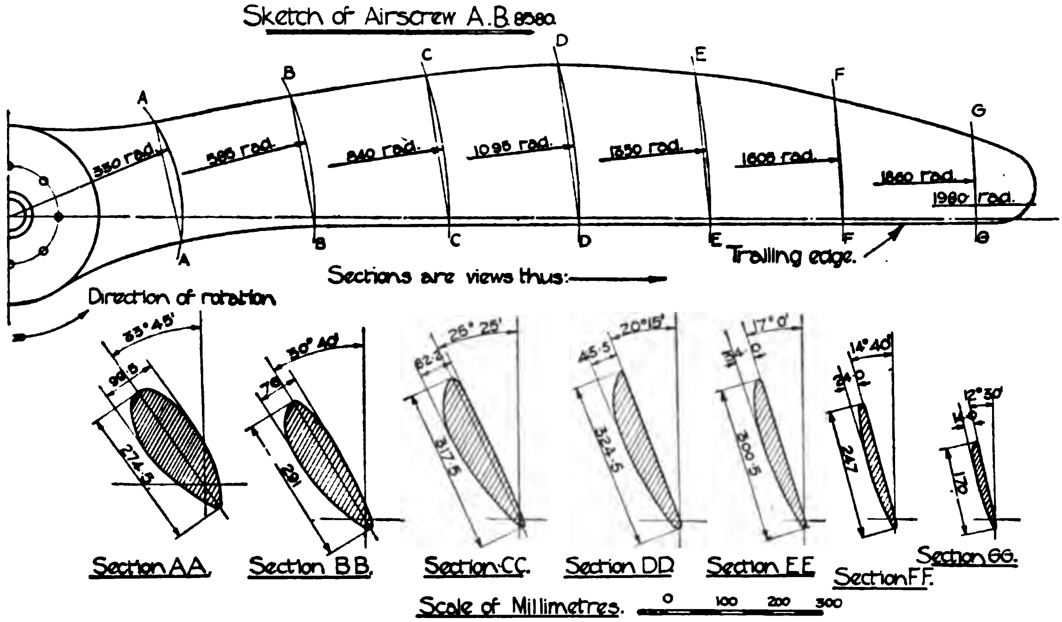

Sketch of Airscrew A.B 8580
330 rad.
585 rad.
840 rad.
1095 rad.
1350 rad.
1605 rad.
1860 rad.
1980 rad.
Trailing edge.
Sections are views thus:
Direction of rotation
33°45'
30°40'
25°25'
20°15'
17°0'
14°40'
12°30'
274.5
291
317.5
324.5
300.5
247
172
Section AA.
Section BB.
Section CC.
Section DD.
Section EE.
Section FF.
Section GG.
Scale of Millimetres.
0 100 200 300

FIG. 71.

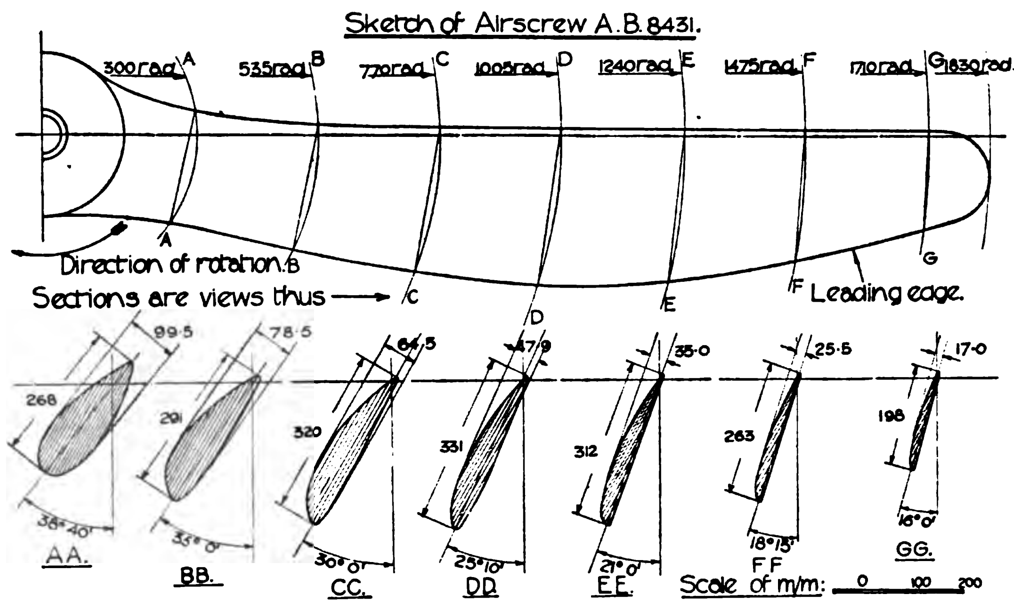

Sketch of Airscrew A.B. 8431.
300 rad.
535 rad.
770 rad.
1005 rad.
1240 rad.
1475 rad.
1710 rad.
1830 rad.
Direction of rotation.
Sections are views thus
Leading edge.
99.5
78.5
64.5
47.9
35.0
25.5
17.0
268
291
320
331
312
263
198
38°40'
33°0'
30°0'
25°10'
21°0'
18°15'
16°0'
AA.
BB.
CC.
DD.
EE.
FF
GG.
Scale of m/m:
0 100 200

FIG. 72.

forward airscrew. The experimental data of these experiments are shown graphically in Figs. 69 and 70.

At the same values of the forward and rotational speeds both the thrust and torque are smaller when an airscrew is working in the outflowing stream from a forward airscrew. This is largely due to the diminution of the angle of incidence of the blade caused by the augmented translational velocity at the front of the airscrew. It will also be noticed that the shifts of the thrust and torque curves of the rear airscrew in the direction of the increasing value of the rotational speed —due to the outflowing stream from the forward airscrew—are much more pronounced when the two airscrews rotate in the same direction. It would follow, then, that when designing the rear airscrew of a tandem combination some allowance should be made for the magnitude and the direction of rotation of the air in the outflowing stream of the forward airscrew as well as for the increased translational velocity. We found from the data of the experiments with the combination, in which the airscrews were of equal diameter rotating in the opposite direction, that when the forward and rear airscrews are developing equal thrusts, the maximum efficiency of the rear airscrew was about 90 per cent of the value when the forward airscrew was not working. At climbing, the efficiency of the rear airscrew was about 85 per cent of the value when the forward airscrew was not working. We also found that the efficiency of a tandem combination with the airscrews rotating in the opposite direction is greater than that with the airscrews rotating in the same direction. With the particular airscrews of which the performance data are given in Fig. 70 the efficiency of the *tandem* combination was increased about 4 per cent by rotating the airscrews in the opposite direction, when the airscrews developed equal thrusts. It is here assumed—from the evidence of these experiments—that the working of the rear airscrew does not appreciably affect the performance of the forward airscrew.

From the experiments made with tandem combinations in which airscrews of different diameter were rotated in the opposite direction we found that the best performance was obtained when the forward airscrew was the larger. Also when engines of unequal horse-power are mounted in a tandem combination it is preferable to mount the forward and larger airscrew on the engine having the higher horse-power.

Some further experiments* with five model airscrews arranged in four tandem combinations were made at the National Physical Laboratory. These experiments, which were not of such a general nature as those just described, were made with models of airscrews designed for the Rolls-Royce Eagle, Series 8, mounted on the Handley-Page Bombing Machine. The Rolls-Royce Eagle develops about 350 h.p. at 1800 r.p.m., the ratio of the reduction gearing being 0·6. With each tandem combination, a wooden shell representing the fairing of the tandem engines was mounted between the two airscrews. It is proposed to describe in some detail the data of these experiments because the airscrews,

* "Some further experiments on tandem airscrews," by A. Fage, A.R.C.SC., and H. E. Collins. With an Addendum by H. C. Watts, B.SC., and Capt. J. G. M. Bevan, B.SC. Advis. Comm. Aeron., 1918.

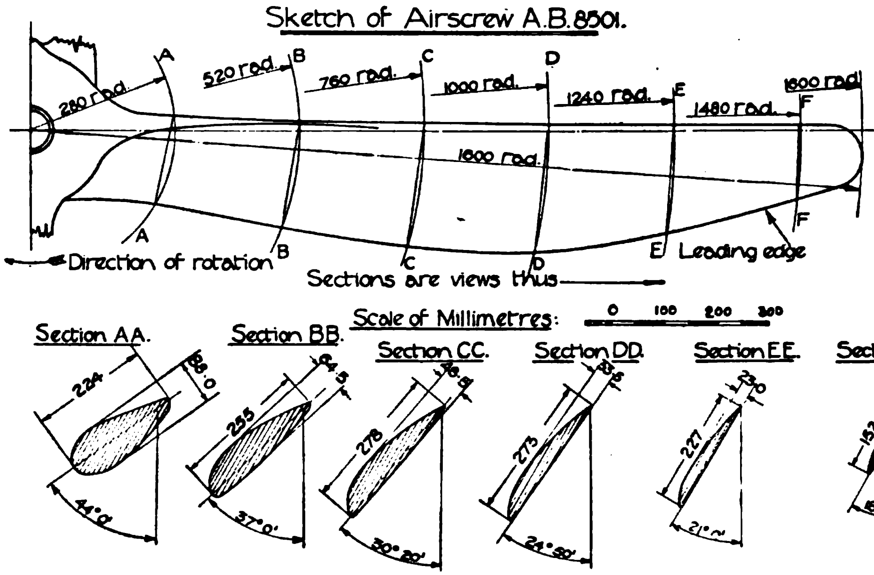

Sketch of Airscrew A.B.8501.
280 rad.
520 rad.
760 rad.
1000 rad.
1240 rad.
1480 rad.
1600 rad.
1600 rad.
Direction of rotation
Leading edge
Sections are views thus
Scale of Millimetres: 0 100 200 300
Section AA.
Section BB.
Section CC.
Section DD.
Section EE.
Section FF.
224
88.0
44°0'
255
64.5
37°0'
278
30°20'
273
24°50'
227
23.0
21°
152
15.0
18°10'

FIG. 73.

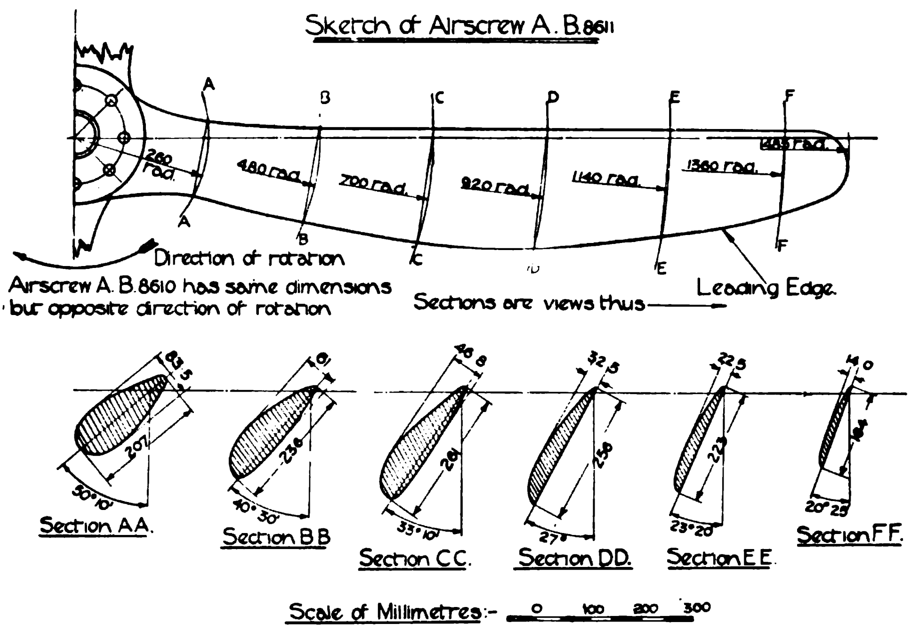

Sketch of Airscrew A. B.8611
280 rad.
480 rad.
700 rad.
920 rad.
1140 rad.
1360 rad.
Direction of rotation
Airscrew A. B.8610 has same dimensions but opposite direction of rotation
Sections are views thus
Leading Edge.
83.5
207
50°10'
61
236
40°30'
46.8
261
33°10'
32.5
258
27°
22.5
223
23°20'
14.0
184
20°25'
Section AA.
Section BB
Section CC.
Section DD.
Section EE.
Section FF.
Scale of Millimetres:- 0 100 200 300

FIG. 74.

which were designed at the Air Ministry by Mr. Watts, are considered to be representative of modern design. Some general characteristics of the five airscrews of which sketches are shown in Figs. 71–74 are collected in Table IX. Experiments were made with the forward airscrew A.B. 8580 working alone, and working in the inflowing stream of each rear airscrew, when the torques of the forward and rear airscrews were equal. Also the performances of each rear airscrew were measured when working alone, and when working in the outflowing stream of, and at the same torque as, the forward airscrew.

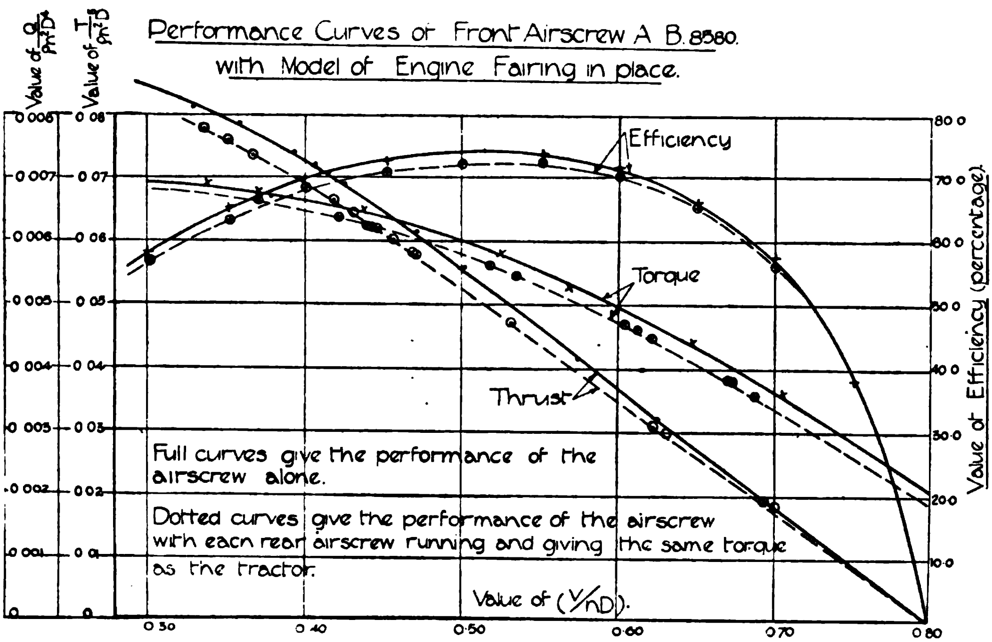
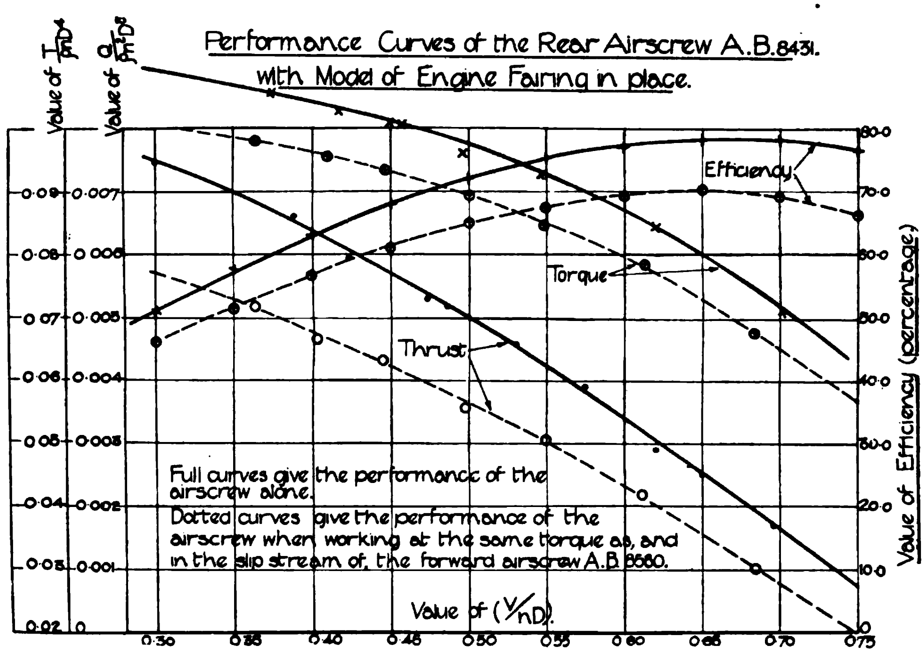


FIG. 75.

TABLE IX

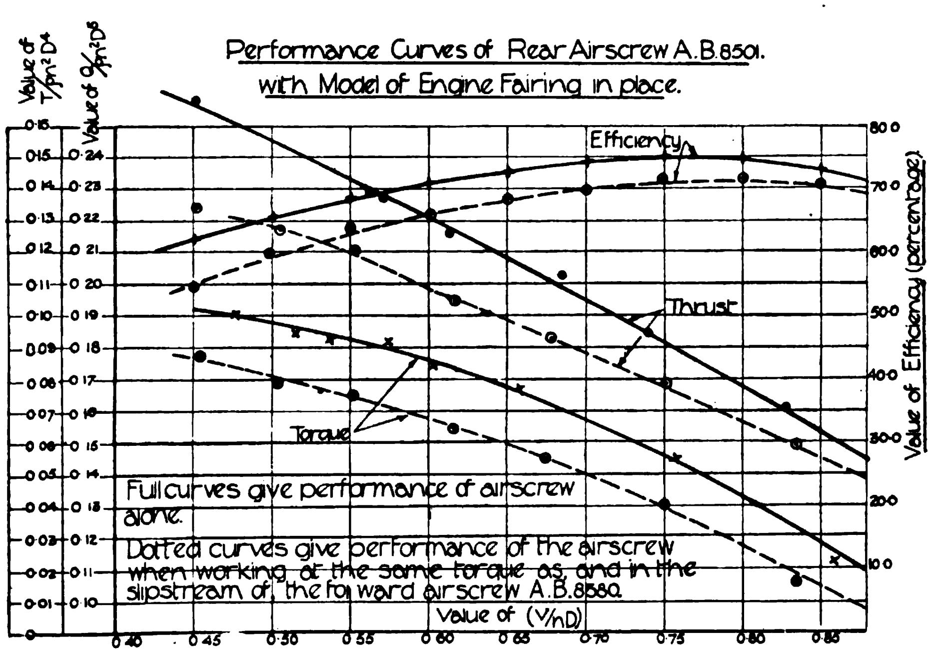
Airscrew.	Number of blades.	Diameter, feet.	Direction of rotation.	Position in tandem combination.
A.B. 8580	2	13 ft. 0 in.	Left-handed	Forward
A.B. 8431	2	12 ft. 0 in.	Right-handed	Rear
A.B. 8501	4	10 ft. 6 in.	Right-handed	Rear
A.B. 8611	4	9 ft. 9 in.	Right-handed	Rear
A.B. 8610	4	9 ft. 9 in.	Left-handed	Rear

With the exception of the direction of rotation, airscrews A.B. 8611 and A.B. 8610 are equal in all respects. The data of the experiments are plotted in Figs. 75–79. From Fig. 75 it is seen that the forward airscrew A.B. 8580 has a



Performance Curves of the Rear Airscrew A.B.8431.
with Model of Engine Fairing in place.
Efficiency
Torque
Thrust
Full curves give the performance of the airscrew alone.
Dotted curves give the performance of the airscrew when working at the same torque as, and in the slip stream of, the forward airscrew A.B 8580.
Value of (V/nD).
Value of Efficiency (percentage)

FIG. 76.



Performance Curves of Rear Airscrew A.B.8501.
with Model of Engine Fairing in place.
Efficiency
Thrust
Torque
Full curves give performance of airscrew alone.
Dotted curves give performance of the airscrew when working at the same torque as, and in the slipstream of, the forward airscrew A.B.8580.
Value of (V/nD)
Value of Efficiency (percentage)

FIG. 77.

maximum efficiency of 74 per cent when working alone. At any value of (V/nD) the efficiency of the forward airscrew when working in a tandem combination, and absorbing the same torque as the rear airscrew, is about 98 per cent of the value when the airscrew works alone. We found that the interference of each rear airscrew on the forward airscrew was approximately of the same magnitude. Generally speaking, when the forward and rear airscrews have opposite directions of rotation the maximum efficiency of the rear airscrew, when working in the tandem combination at the same torque as the forward airscrew, is about 93 per cent of its value when working alone. From a comparison of the data of experiments made with airscrews A.B. 8610 and A.B. 8611 it would appear undesirable to rotate both airscrews of a tandem combination in the same direction.

A comparison at the same value of (V/nD) of the efficiencies of an airscrew—with and without the interference of the other airscrew of the tandem combination—may be somewhat misleading because the horse-powers absorbed may differ. Accordingly, then, comparisons of theoretical interest, which are of some value as showing the modification of the performance of an airscrew when working in a tandem combination, should be made when the airscrews are regarded as separate entities absorbing the same horse-power at the same forward speed, adjustment being made, if necessary, on both the torque and the rotational speed. From the experimental data of the five airscrews, calculations of the efficiency have been made for two conditions of working, namely when each airscrew absorbs (1) 320 h.p. at a climbing speed of 55 m.p.h. and (2) 350 h.p. at a forward speed of 95 m.p.h. The data of these calculations are given in Table X. It will there be seen that at forward speeds of 95 m.p.h. and 55 m.p.h. the efficiencies of the forward airscrew when working in the tandem combination are 99 per cent and 97·5 per cent respectively of the values when the airscrew is working alone. The most efficient rear airscrew would appear to be A.B. 8501. With an opposite direction of rotation the efficiency of a rear airscrew when working in the combination is about 87·5 per cent at the climb, and 92 per cent at the forward speed of 95 m.p.h., of the corresponding values for the airscrew working alone. With the same direction of rotation the efficiency of a rear airscrew is greatly reduced when working in a tandem combination. With airscrew A.B. 8610 the fall of efficiency is as great as 81·5 per cent at the climbing speed of 55 m.p.h. and 89·5 per cent at the forward speed of 95 m.p.h.

Of the three rear airscrews which rotate in the opposite direction to that of the forward airscrew, the reduction of efficiency due to the interference of the forward airscrew is greatest for the airscrew of largest diameter, from which it would appear—although the evidence is not of a very conclusive nature—that there is some advantage in mounting an airscrew of small diameter at the rear, when engines of equal power are mounted in a tandem combination.

TABLE X

Comparison of the Performance of each Airscrew when Working Alone and when Working in the Tandem Combination

The comparisons are made when the airscrew, whether working alone or in the tandem combination, absorbs the same horse-power at the same forward speed of the machine.

Airscrews tested with interference of engine fairing.

Airscrew.	Remarks.	Airscrew alone.		Airscrew in tandem.		Ratio of efficiencies. (B/A)	
		Efficiency percentage. Column A.	Rotational speed, r.p.m.	Efficiency percentage. Column B.	Rotational speed, r.p.m.		
A.B. 8580* L.H.	Airscrew absorbing	66·8	1008	65·0	1016	0·975	
A.B. 8431 R.H.	320 h.p. at a climb-	60·5	1060	52·0	1100	0·860	
A.B. 8501 R.H.	ing speed of 55	62·3	1015	54·5	1045	0·875	Mean = 0·875
A.B. 8611 R.H.	m.p.h. at ground	58·3	1041	52·0	1056	0·890	
A.B. 8610 L.H.	level.	61·5	1041	50·0	1115	0·815	
A.B. 8580* L.H.	Airscrew absorbing	72·8	1120	72·0	1134	0·990	
A.B. 8431 R.H.	350 h.p. at a for-	76·3	1200	68·5	1210	0·900	
A.B. 8501 R.H.	ward speed of 95	74·5	1121	70·0	1155	0·940	Mean = 0·920
A.B. 8611 R.H.	m.p.h. at ground	74·0	1138	68·0	1160	0·920	
A.B. 8610 L.H.	level.	77·3	1133	69·0	1195	0·895	

* Forward airscrew

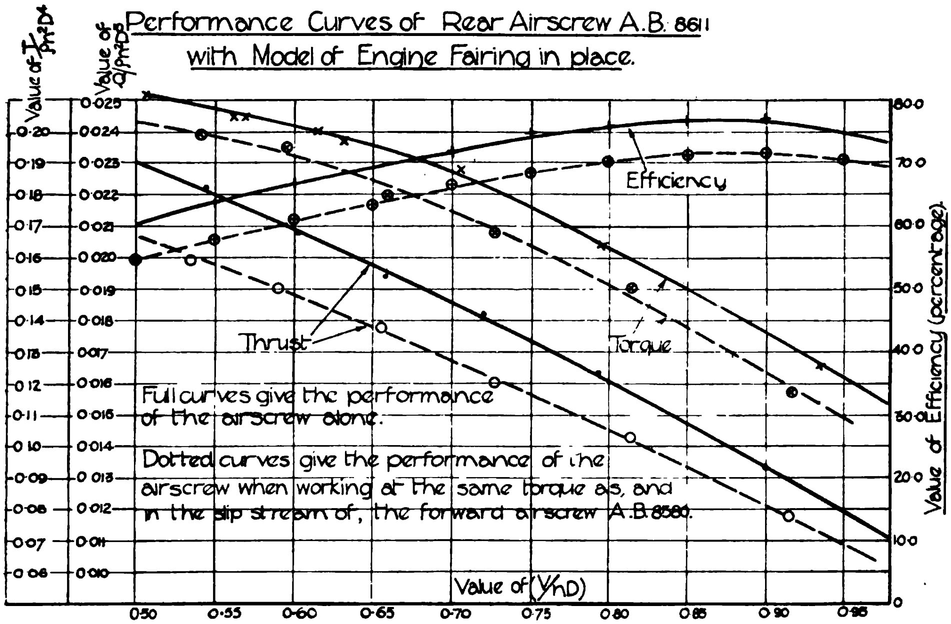

Performance Curves of Rear Airscrew A.B. 8611
with Model of Engine Fairing in place.
Efficiency
Thrust
Torque
Full curves give the performance of the airscrew alone.
Dotted curves give the performance of the airscrew when working at the same torque as, and in the slip stream of, the forward airscrew A.B.8580.
Value of (V/nD)
Value of Efficiency (percentage)
0·50
0·55
0·60
0·65
0·70
0·75
0·80
0·85
0·90
0·95
80·0
70·0
60·0
50·0
40·0
30·0
20·0
10·0
0

FIG. 78.

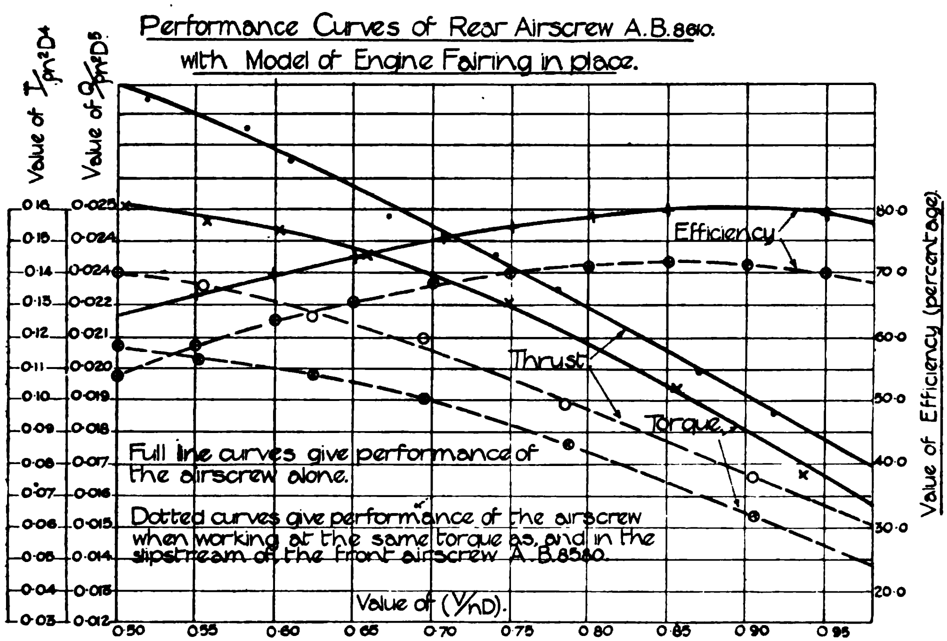

Performance Curves of Rear Airscrew A.B.8610.
with Model of Engine Fairing in place.
Efficiency
Thrust
Torque
Full line curves give performance of the airscrew alone.
Dotted curves give performance of the airscrew when working at the same torque as, and in the slipstream of the front airscrew A.B.8580.
Value of (V/nD).
Value of Efficiency (percentage)
0·50
0·55
0·60
0·65
0·70
0·75
0·80
0·85
0·90
0·95
80·0
70·0
60·0
50·0
40·0
30·0
20·0

FIG. 79.

Calculation of the Mean Translational and Rotational Velocities of the Outflowing Stream of an Airscrew

Assuming that the only effect of the forward airscrew is to increase the translational motion uniformly, and to impart a uniform rotational motion to the air within the outflowing stream, it is possible to calculate the magnitudes of both these velocities from the performance data of two equal rear airscrews of opposite direction of rotation. It is now proposed to calculate from the experimental data of airscrews A.B. 8610 and A.B. 8611—working alone and also in the tandem combination—the magnitudes of the translational velocity, and the spin of the air in the outflowing stream, of the forward airscrew A.B. 8580.

When testing a rear airscrew, the speed relative to the air outside the disturbance of the tandem combination, and the actual speed of rotation are measured. Also from the theoretical standpoint, a rear airscrew will be giving the same thrust—and the same torque—when the translational and rotational motions relative to the air undisturbed by the airscrew itself remain unchanged. If, then, V and n are the measured translational and rotational speeds at which either the left-handed airscrew A.B. 8610 or the right-handed airscrew A.B. 8611, when working alone, develops a given thrust—or absorbs a given torque—and V_1, V_2 are the translational speeds, and n_1, n_2 the rotational speeds respectively at which the same thrust is developed—or the same torque absorbed—when working in the tandem combinations, then

$$V = V_1 + abV = V_2 + abV, \text{ that is } V_1 = V_2 \text{ and } n = n_1 - n_f = n_2 + n_f,$$

$$\text{that is } n = \left(\frac{n_1 + n_2}{2}\right) \text{ and } n_f = \left(\frac{n_1 - n_2}{2}\right),$$

where abV is the augmented translational velocity of the air in the outflowing stream from the forward airscrew, and n_f the mean value of the imparted rotational speed.

It is now proposed to illustrate, by an arithmetical example, the method of calculating the speeds of both the rotational and translational motions in the outflowing stream of the forward airscrew. Before doing so, however, it is necessary to apply appropriate corrections to the experimental data of airscrews A.B. 8610 and 8611 (including the performances in the tandem combination) to make the performances of these two airscrews equal when working alone. The experimental data of Figs. 78 and 79 when finally corrected are given in Table XI. The discrepancy between the experimental data—which was not very appreciable—was probably due to the great difficulty of constructing two exactly equal airscrews.

At a translational speed of 120 ft. per sec. and a rotational speed of 17·5 r.p.s. the thrust and torque when either airscrew A.B. 8610 or airscrew A.B. 8611 is working alone—as calculated from the data of Table XI—are 980 lb. and 1460 lb.-ft. respectively. With airscrews A.B. 8610 and A.B. 8611 working in the tandem combinations, the same thrust is developed at rotational speeds of 18·34 r.p.s. and 16·66 r.p.s. respectively, the translational speed in each case being 89 ft. per sec. The mean value of these rotational speeds is 17·5 r.p.s.

TABLE XI

Performance Data of Airscrews A.B. 8610 and A.B. 8611

Airscrews A.B. 8610 and A.B. 8611 are equal but of opposite direction of rotation. The experimental data of these two airscrews when each is working alone should be equal. The table gives performance data after small corrections have been made for slight differences of blade shape.

V/nD	Airscrew working alone. Mean of experimental data for airscrews A.B. 8610 and A.B. 8611.		Airscrew working in tandem combination. Corrected performance.			
			Airscrew A.B. 8610.		Airscrew A.B. 8611.	
	$T/\rho V^2D^2$	$Q/\rho V^2D^3$	$T/\rho V^2D^2$	$Q/\rho V^2D^3$	$T/\rho V^2D^2$	$Q/\rho V^2D^3$
0·95	0·0925	0·0182	0·0695	0·0158	0·0815	0·0167
0·90	0·1205	0·0219	0·0905	0·0188	0·1050	0·0204
0·80	0·1945	0·0320	0·1460	0·0268	0·1700	0·0302
0·70	0·3060	0·0465	0·2255	0·0385	0·2720	0·0442
0·60	0·4830	0·0675	0·3510	0·0552	0·4260	0·0647
0·50	0·7800	0·1010	0·5400	0·0826	0·6880	0·0975

It follows, then, that the augmented translational velocity, abV, of the air in the outflowing stream of the forward airscrew is (120 − 89), that is 31 ft. per sec. Also the value of the mean spin is $\left(\frac{18\cdot34-16\cdot66}{2}\right)$, that is 0·84 r.p.s.

Making a similar calculation for constancy of torque (Q = 1460 lb.-ft.), it was found that with each airscrew working in a tandem combination $V_1 = 83$ ft. per sec. and $n_1 = 18\cdot1$ r.p.s. for airscrew A.B. 8610, also $V_2 = 83$ ft. per sec., and $n_2 = 16\cdot9$ r.p.s. for airscrew A.B. 8611.

The mean of the rotational speeds is 17·5 r.p.s.

Hence abV = 37 ft. per sec. and $n_r = 0\cdot6$ r.p.s.

The discrepancies between the two values of abV and of n_r are probably due to the inaccuracies of the assumptions that the effects of the working of the forward airscrew are to increase uniformly the translational velocity and to impart a uniform angular motion to the air dealt with by the airscrew. The mean of the two values of abV and n_r as calculated above are 34 ft. per sec. and 0·72 r.p.s., so that the velocity of the forward airscrew relative to the undisturbed air in front is (120–34), that is 86 ft. per sec., when the forward and rear airscrews are each absorbing a torque of 1460 lb.-ft. The corresponding values of the thrust and of the rotational speed of the forward airscrew are 1190 lb. and 16·15 r.p.s. It follows, then, that with this condition of working—a very low climbing value of (V/nD)—the ratio of the mean spin to the rotational speed of the forward airscrew is about 4·5 per cent. When designing the rear airscrew of a tandem combination, allowance should therefore be made for the rotational motion in the outflowing stream of the forward airscrew as well as for the augmented translational motion.

A Practical Analysis of some Experimental Data of Tandem Airscrews

We have so far considered only the performance of an airscrew apart from that of the engine. The important consideration is, of course, the overall efficiency of the combination of airscrew and engine; it is desirable that an airscrew should have not only a good efficiency, but that the engine should be working at its most economical speed. The addendum previously mentioned was prepared by Mr. Watts with the object of analysing the experimental airscrew data in conjunction with the characteristics of the engine on which the airscrews were mounted.

The performance data of this engine, namely the Rolls-Royce Eagle, are given in Table XII.

From the performance data of the engine and each airscrew, calculations were made which show how the thrust horse-power and rotational speed of each airscrew, whether working alone or in the tandem combination, vary with the forward speed of the aeroplane. It was found that at any forward speed the airscrews would not rotate at the same speed and therefore the thrust horse-power could not be compared with strict accuracy, since different horse-powers were absorbed

by the several airscrews. To make a true comparison the performances of the airscrews were reduced to the standard running conditions of the design, by assuming each airscrew to be mounted on an engine rotating at 1900 r.p.m. at a forward speed of 100 m.p.h., the engine having a torque curve of the same shape as the standard engine. The data of such calculations are shown graphically in Figs. 80 and 81. From the curves of these figures it is seen that the decrease of rotational speed with a decrease of forward speed is not the same with each airscrew. In order that a practical comparison of the several airscrews may be made it is necessary to know how the thrust horse-power, which is a function of

TABLE XII

Ratio of Reduction Gearing 0·6

Rotational speed of engine, r.p.m.	Torque of engine, lb.-ft.
2100	1560
2000	1620
1900	1665
1800	1705
1700	1740
1600	1775
1500	1810

both efficiency and the horse-power absorbed, varies with the forward speed of the aeroplane. Accordingly, then, calculations of "overall" efficiency were made, where the

$$\left(\begin{array}{c}\text{“ Overall ” effi-}\\ \text{ciency at any}\\ \text{forward speed}\end{array}\right) = \left(\begin{array}{c}\text{Airscrew effi-}\\ \text{ciency at that}\\ \text{speed}\end{array}\right) \times \left(\frac{\begin{array}{c}\text{Engine h.p. at the r.p.m. at}\\ \text{this forward speed}\end{array}}{\begin{array}{c}\text{Engine h.p. at the r.p.m. at the}\\ \text{maximum speed of 100 m.p.h.}\end{array}}\right)$$

The data of these calculations are collected in Table XIII, from which it is seen that the average losses of thrust horse-power of a rear airscrew when working in the tandem combination are 7·5 per cent at top speed, and 9·0 per cent at the climb, of the corresponding values when the airscrew is working alone. With the forward airscrew, the losses are 0 per cent and 3 per cent at the maximum forward speed and the climb respectively. The climbing speed is here assumed to be 0·7 of the maximum forward speed.

Assuming the above losses at the maximum forward speed and the climb to be typical with two similar engines mounted in tandem, Watts made the following general conclusion. The thrust horse-power given out by a power plant with two similar engines working in tandem is 1·93 and 1·88 at top speed and climb

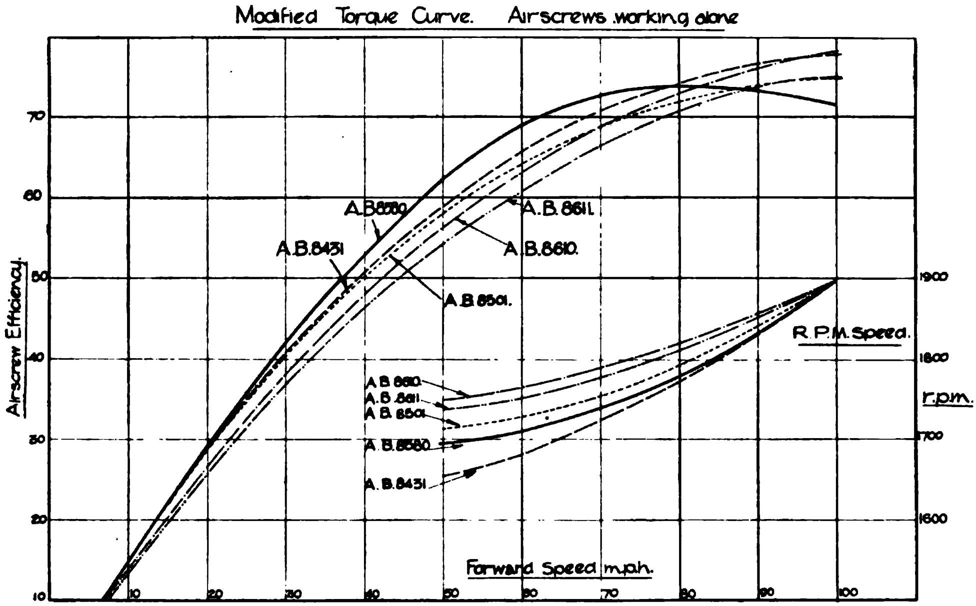

Modified Torque Curve. Airscrews working alone
Airscrew Efficiency.
A.B.8580
A.B.8431
A.B.8611.
A.B.8610.
A.B.8501.
R.P.M. Speed.
r.p.m.
A.B.8610
A.B.8611
A.B.8501
A.B.8580
A.B.8431
Forward Speed m.p.h.

Fig. 80.

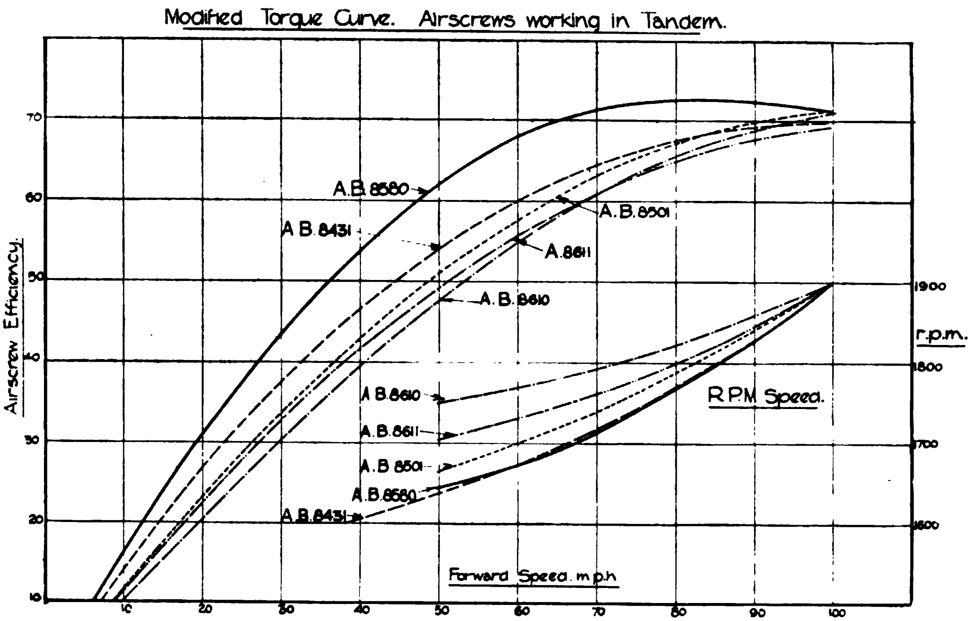

Modified Torque Curve. Airscrews working in Tandem.
Airscrew Efficiency.
A.B.8580
A.B.8431
A.B.8501
A.8611
A.B.8610
r.p.m.
R.P.M. Speed.
A.B.8610
A.B.8611
A.B.8501
A.B.8580
A.B.8431
Forward Speed m.p.h.

Fig. 81.

respectively of the thrust horse-power given by a power plant consisting of only the tractor engine. This general conclusion may be taken as sufficiently accurate for purposes of estimation, although it should be remembered that the comparison has been made between the performance of the same airscrew when working alone and when working in tandem, whereas the comparison should be made between the best airscrew for the condition of working alone and the best airscrew for the conditions of the tandem.

TABLE XIII

	Speed.	Overall efficiency working alone. (A)	Overall efficiency working in tandem. (B)	Value of Column B / Value of Column A
Airscrew 8580 (Forward)	100	71·6	71·8	1·000
	90	72·0	71·1	1·000
	80	71·5	69·6	0·975
	70	69·0	67·0	0·970
	60	65·0	62·9	0·970
	50	59·5	57·0	0·960
Airscrew 8431 (Rear)	100	78·3	70·0	0·895
	90	75·8	68·1	0·900
	80	71·9	65·1	0·905
	70	67·1	61·0	0·900
	60	61·3	55·8	0·910
	50	54·3	49·2	0·905
Airscrew 8501 (Rear)	100	75·1	71·9	0·955
	90	73·1	69·0	0·940
	80	70·0	65·0	0·930
	70	66·0	60·1	0·910
	60	60·3	54·1	0·900
	50	53·6	47·0	0·895
Airscrew 8611 (Rear)	100	75·5	69·7	0·925
	90	72·9	67·0	0·920
	80	69·1	63·6	0·910
	70	64·2	58·2	0·905
	60	58·4	52·3	0·895
	50	51·1	45·2	0·885

CHAPTER IX

THE STRESSES AND DISTORTIONS OF AN AIRSCREW BLADE UNDER LOAD

The Working Stresses of an Airscrew

It is of some importance to know the magnitude of the working stresses of an airscrew blade, in view of the large horse-powers which are absorbed in some modern airscrews. An exact solution of the problem is unfortunately a matter of extreme difficulty, especially when consideration is taken of the heterogeneity of the physical properties of the material from which the airscrew is constructed. In a complete investigation of the working stresses of an airscrew consideration would need to be given to (*a*) the aerodynamic loading, of which the magnitude is a function of the speed and the attitude with which the blades meet the air, (*b*) the centrifugal loading, which depends on the rotational speed, shape, and size of the airscrew, and (*c*) the inertia loading due to any unsteady motion of the aeroplane, e.g. linear and rotational accelerations and gyroscopic vibrations. In the present chapter we shall regard the aeroplane to be in steady rectilinear flight, so that only the aerodynamic and centrifugal loadings are considered. If the blade of the airscrew be regarded as a cantilever, the resultant force at any section, due to the loading on the outer part of the blade, can be completely expressed by three forces and three couples defined with reference to rectangular axes.

Such a system of rectangular axes is shown in Fig. 82, where OX passes through the C.G. of the section and is parallel to the chord, and OY taken in the plane of the blade section is perpendicular to OX. The axis of OZ is, of course, at right angles to both OX and OY. The direction of rotation is given by OA and the direction of translation by OC. The resultant force at any section, due to the loading on the outer part of the blade, can be completely expressed by three forces and couples, defined with reference to rectangular axes.

The aerodynamic loading at the blade section may be completely represented by—

(*a*) A bending moment, M_T, about the axis OA due to the thrust.

(*b*) A bending moment, M_Q, about the axis OC due to the torque.

(*c*) A twisting moment, M_A, about the axis OZ due to the resultant of the air force on the outer part of the blade not passing through OZ. The twisting moment M_A is difficult to calculate but can be made small by suitably designing the shape of the blade.

(*d*) A shearing force, F_A, across the section. This shearing force is probably

small, but it is worth noting that as a consequence of F_A there is a shearing stress in the plane of the glue joints.

(*e*) An air force acting at right angles to the section. This force is due to any radial air-flow along OZ, and is the result of any departure from a two-dimensional air-flow. No further account need be taken of this force, which is very small.

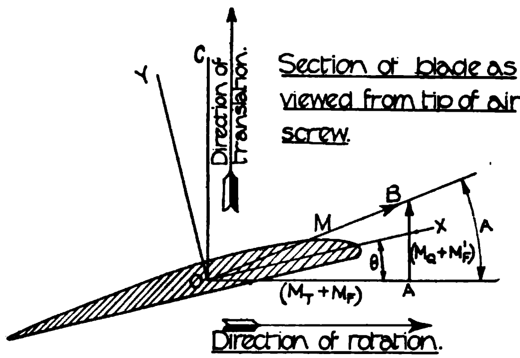


FIG. 82.

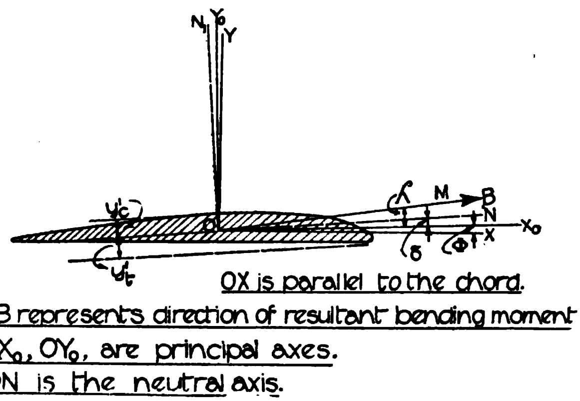


FIG. 83.

If the line passing through the C.G. of the outer portion of the blade at right angles to the axis of rotation does not coincide with the radial line OZ the centrifugal loading at the section may be completely defined by—

(*a*) Bending moments M_F and M'_F about the axes OA and OC respectively, and a twisting moment about OZ. The twisting moment will always be small—especially for the sections at the tip, and can be neglected.

(*b*) A tensile force F acting along OZ.

(*c*) A shearing force, F_c, acting across the section. This shearing stress is small and may be neglected.

Neglecting the shearing forces F_A and F_c and also the twisting moment M_A, we may regard the loading at a blade section to be equivalent to a bending moment (M_T+M_F) about the axis OA, a bending moment $(M_Q+M'_F)$ about the axis OC, and a tensile force F acting at right angles to the section.

The resultant bending moment M on the section is therefore

$$\sqrt{(M_T+M_F)^2+(M_Q+M'_F)^2},$$

the axis of this moment making an angle A with OA where

$$\tan A = \frac{(M_Q+M'_F)}{(M_T+M_F)}.$$

A Method of Calculating the Working Stresses of an Airscrew Blade

A simple method of calculating the stresses of an airscrew blade, which are due to both the bending moment $\sqrt{(M_T+M_F)^2+(M_Q+M'_F)^2}$ and the centrifugal force, F, at the section, is now given. It should perhaps be stated that this

method, which is commonly employed in the Drawing Office, was first brought to the author's notice by Bolas.*

In this case the stresses are calculated from the component of the resultant bending moment about the axis through the centre of gravity of the section parallel to the chord. Hence the maximum compressive stress

$$=\left[\frac{M.\cos(A-\theta).y_c}{I_C}-\frac{F}{S_B}\right]$$

and the maximum tensile stress

$$=\left[\frac{M.\cos(A-\theta)y_t}{I_c}+\frac{F}{S_B}\right],$$

where S_B = area of the blade section,

I_c = moment of inertia of the blade section about an axis through the C.G. parallel to the chord, and

$$M=\sqrt{(M_Q+M'_F)^2+(M_T+M_F)^2}.$$

Also y_c and y_t are the maximum distances of points on the periphery of the section from the axis about which the moment of inertia I_c is calculated, y_c being

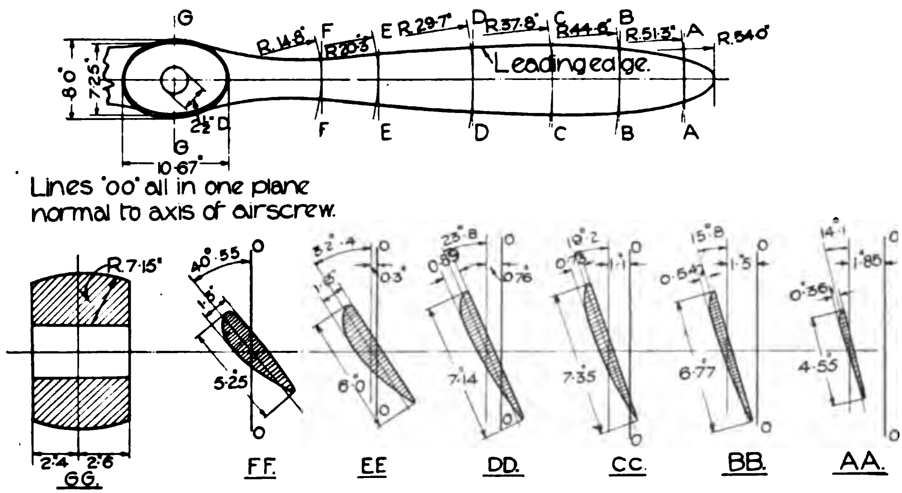


FIG. 84.

measured on the left-hand side of the axis of the bending moment M $\cos(A-\theta)$, looking in the direction of the arrow, shown in Fig. 82. To illustrate the preceding method it is now proposed to calculate the working stresses of the two-bladed airscrew of which a sketch is given in Fig. 84.

This airscrew has a diameter of 9 ft., the blade angle varying from 14·1° at the tip to 40·55° at the section next to the boss. The blade is so shaped that the tip sections as viewed from behind are lifted forward relatively to the boss, so that the bending moment at any section due to the thrust is partly balanced by a bending moment due to the centrifugal force acting on the outer part of the

* "Notes on aerial propellers," by H. Bolas. Advis. Comm. Aeron., 1912.

blade. The centre of gravity of each blade section is in a plane passing through the axis of rotation, so that the bending moment M'_F about the axis OC and also the twisting moment about the axis OZ—both due to centrifugal force—are each zero. With this design of airscrew the twisting moment M_A about OZ at any section of the blade due to the air force acting on the outer part of the blade is probably small at the working speeds of the airscrew, and has been neglected in the present calculation.

A collection of data calculated from the shape of the blade sections is given in Table XIV.

If $c \equiv$ length of the chord of a blade section.

$\sigma c \equiv$ maximum thickness of the section.

$S_B \equiv$ area of the section.

$I_C \equiv$ moment of inertia of the section about the axis OX through the C.G. parallel to the chord.

$I_N \equiv$ moment of inertia of the section about the axis OY through the C.G. at right angles to the chord.

$D \equiv$ product of inertia of the section about these axes.

$\bar{Z} \equiv$ height of the C.G. of the section above the chord.

Then with good accuracy

$$S = 0{\cdot}72\sigma c^2.$$
$$I_C = 0{\cdot}049\sigma^3 c^4.$$
$$I_N = 0{\cdot}043\sigma c^4.$$
$$\bar{Z} = 0{\cdot}40\sigma c.$$
$$D = 0{\cdot}01\sigma^2 c^4.$$

So that if I_{θ_1} represents the moment of inertia of the section about any line passing through the C.G. and making an angle θ_1 with the chord, then

$$I_{\theta_1} = 0{\cdot}049\sigma^3 c^4 \cos^2\theta_1 + 0{\cdot}043\sigma c^4 \sin^2\theta_1 - 0{\cdot}01\sigma^2 c^4 \sin 2\theta_1.$$

Calculations of stress are made from the condition of loading at the maximum horizontal flight speed of 95 m.p.h., the rotational speed of the airscrew being 1375 r.p.m., the thrust 350 lb., and the torque 470 lb.-ft.

The Calculation of the Values of M_T, M_Q, M_F, and F

The thrust-grading curve for the airscrew blade at the forward speed of 95 m.p.h. is given in Fig. 85. The thrust-grading curve gives the intensity of loading due to the thrust, $(\delta T/\delta r)$, and is constructed from the aerodynamic data of the blade sections. The thrust on the part of the blade outside the radius, r, is $\int_r^R \frac{\delta T}{\delta r} dr$, and is directly obtained from an integration of the thrust-grading curve (see Fig. 86.) In a similar manner, since $M_T = \int_r^R T dr$, the bending moment of any point of the blade is obtained from the integration of the curve of Fig. 86. The bending-moment curve is given in Fig. 87. The bending moment due to the torque M_Q may be obtained in a similar manner, although at the outset a load-

TABLE XIV

Name of section.	Distance of section from airscrew axis, r (inches).	Type of section.	Value of "x" (inches).	Chord length c (inches).	Maximum thickness σc (inches).	Area S_B (square ins.).	Height of C.G. of the section above chord $\bar{Z}$.	Moment of inertia I_C of section about axis through C.G. parallel to chord.	Moment of inertia I_N of section about axis through C.G. at right angles to chord.	Angle of blade section (degrees).
AA	51·3	F	1·85	4·55	0·36	$0{\cdot}73\sigma c^2$	$0{\cdot}415\sigma c$	$0{\cdot}051\sigma^3 c^2$	$0{\cdot}043\sigma c^4$	14·1
BB	44·6	F	1·50	6·75	0·54	$0{\cdot}72\sigma c^2$	$0{\cdot}400\sigma c$	$0{\cdot}047\sigma^3 c^4$	$0{\cdot}043\sigma c^4$	15·8
CC	37·8	F	1·10	7·35	0·73	$0{\cdot}71\sigma c^2$	$0{\cdot}405\sigma c$	$0{\cdot}049\sigma^3 c^4$	$0{\cdot}043\sigma c^4$	19·2
DD	29·7	F	0·76	7·15	0·89	$0{\cdot}72\sigma c^2$	$0{\cdot}400\sigma c$	$0{\cdot}049\sigma^3 c^4$	$0{\cdot}043\sigma c^4$	23·8
EE	20·3	F	0·30	6·00	1·13	$0{\cdot}71\sigma c^2$	$0{\cdot}400\sigma c$	$0{\cdot}047\sigma^3 c^4$	$0{\cdot}043\sigma c^4$	32·4
FF	14·8	C_S	0·00	5·25	1·30	$0{\cdot}71\sigma c^2$	$0{\cdot}425\sigma c$	$0{\cdot}043\sigma^3 c^4$*	$0{\cdot}042\sigma c^4$	40·6

F = Flat under-surface. C_S = Convex under-surface.

* This height is taken above a line touching bottom surface and parallel to the chord.

Calculation of the Bending Moment due the Thrust.

Translational speed of the airscrew 95 miles/hour.

Rotational speed of the airscrew 1375 r.p.m.

Thrust of the airscrew = 350 lbs.

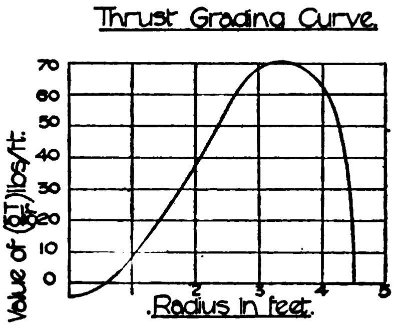


FIG. 85.

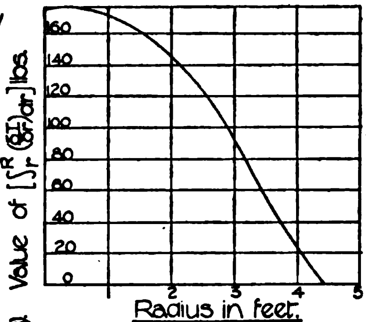


FIG. 86.

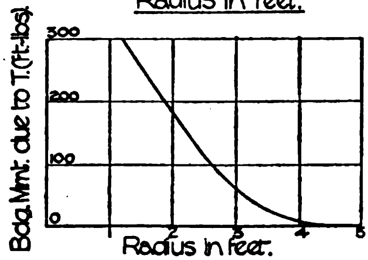


FIG. 87.

Calculation of Bending Moment due to the Torque.

Translation Speed of Airscrew = 95 m.p.h.

Rotational Speed of Airscrew = 1375 r.p.m.

Torque of Airscrew = 470 lbs.-ft.

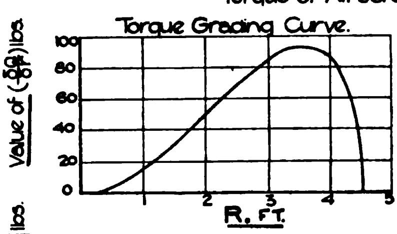


FIG. 88.

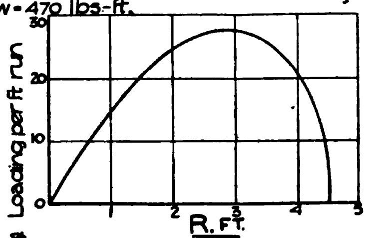


FIG. 89.

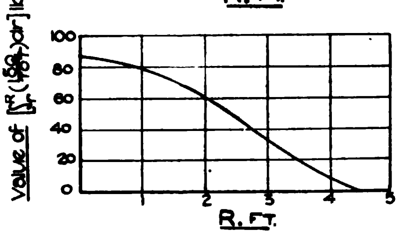


FIG. 90.

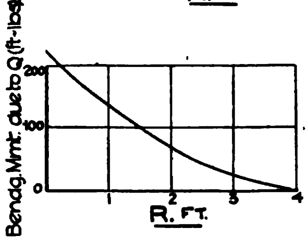


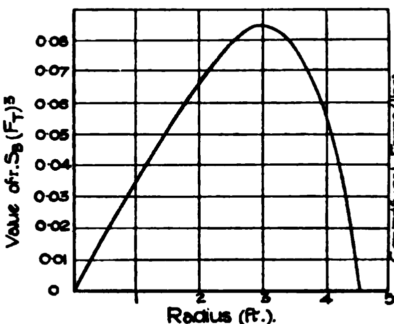
FIG. 91.

intensity curve (Fig. 89) should be constructed from the torque-grading curve (Fig. 88). The diagram of M_Q is given in Fig. 91.

The centrifugal force at any section distance r feet from the airscrew axis $=\int_r^R S_B.r.dr.\Delta(2\pi n)^2$, where n r.p.m. is the rotational speed and Δ is the mass density of wood. In the present paper Δ is assumed to have the value of 42/32·2 slugs per cubic foot.

Hence the centrifugal force, F lb., at the section distance r from the axis of rotation $=0{\cdot}01425\ n^2\int_r^R S_B r dr$. The curve of Fig. 92 gives the value of $S_B r$ plotted against r, from which the values of F have been calculated (see Fig. 93.) The

Calculation of Centrifugal Force.

Fig. 92.


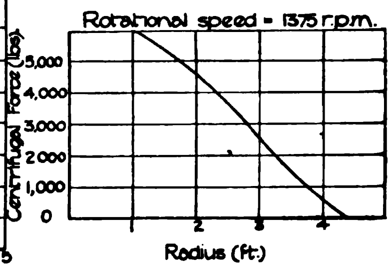


Fig. 93.

Calculation of M_F at section R = 1·23 ft.

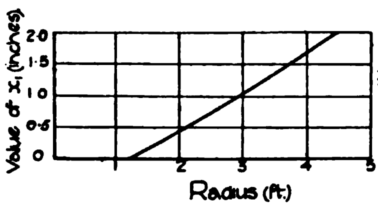


Fig. 94.

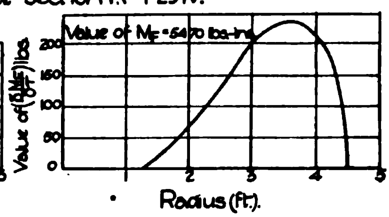


Fig. 95.

bending moment M_F lb.-in., at a section, due to the centrifugal force acting on the outer part of the blade $= -0{\cdot}01425n^2\int_r^R S_B.r.dr.x_1$,

where x_1 is the distance, in inches, of the centre of gravity of a section, taken between the radius r and the blade tip, measured from the plane passing through the centre of gravity of the section at radius r at right angles to the axis of the airscrew. The method of calculating the value of M_F for the section r = 1·23 ft. is illustrated by the curves of Figs. 94 and 95 and Table XV.

TABLE XV

CALCULATION OF BENDING DUE TO THE CENTRIFUGAL FORCE AT THE SECTION r = 1·23 ft.

Rotational Speed = 1375 r.p.m.

Distance of element from axis of airscrew, feet.	$\left(\frac{dF}{dr}\right)$ (lb./ft.)	Value of x_1, inches.	$\left(\frac{dM_F}{dr}\right)$ lb.
1·23	1115	0	0
1·50	1380	0·15	17
1·69	1530	0·25	32
2·00	1785	0·45	67
2·25	1975	0·59	97
2·48	2130	0·73	130
2·75	2240	0·90	168
3·15	2250	1·12	210
3·50	2080	1·35	234
3·72	1840	1·50	230
3·90	1620	1·60	216
4·10	1350	1·75	197
4·28	960	1·85	148
4·50	0	2·00	0

The calculated values of M_T, M_F, M_Q at several blade sections are given in Table XVI. The calculated values of F at the same sections may be directly obtained from the data of Fig. 93. Proceeding in the manner outlined, it is now a simple matter to calculate the maximum compressive and tensile stresses at each section of the blade. The data of these calculations are collected in Table XVI. It will be seen from the table that with this particular airscrew, the bending moment at any section due to the centrifugal force acting on the outer part of the blade is quite large compared with the bending moment due to the air load. Fortunately, however, these bending moments are of opposite sign, so that the stresses due to the resultant bending moment are smaller than those due to the bending moment of the air forces taken alone. It is important, therefore, to calculate the bending moment due to the centrifugal force acting on the outer part of the blade, especially when this bending moment acts in the same direction as that due to the air load, which is the case when the tip of the blade is inclined backwards relative to the boss of the airscrew.

TABLE XVI

STRESSES IN THE AIRSCREW AT THE HORIZONTAL FLIGHT CONDITIONS OF THE AEROPLANE

Translational speed of airscrew =95 m.p.h.
Rotational speed of airscrew =1375 r.p.m.
Thrust=350 lb. Torque=470 lb.-ft.

Distance of blade section from airscrew axis r.ft	M_T in.-lb.	M_F in.-lb.	M_Q in.-lb.	M in.-lb.	A^0	I_c (inch)^4.	y_c inch.	y_t inch.	Stress due to bending moment M. lb. per sq. inch. Compressive. f_c.	Tensile. f_t.	Stress due to centrifugal force (F/S_B) lb. per sq. inch.	Maximum stresses due to bending and the direct centrifugal force. lb. per sq. inch. Compressive.	Tensile.
3·72	228	−179	60	78	50·7	0·050	0·31	0·23	395·0	290·0	370·0	25·0	660·0
3·15	660	−745	216	232	111·5	0·140	0·30	0·43	20·0	28·5	570·0	−550·0	600·0
2·48	1464	−1940	528	711	132·0	0·246	0·35	0·54	315·0	490·0	795·0	−480·0	1300·0
1·69	2796	−4100	1080	1694	140·3	0·406	0·45	0·68	375·0	870·0	1065·0	−700·0	1950·0
1·23	3648	−5470	1440	2330	141·7	0·497	0·56	0·74	485·0	640·0	1185·0	−700·0	1820·0

y_c and y_t are the maximum distances, measured from the axis through the C.G. parallel to the chord, of points on the periphery of the section, y_c being measured on the left-hand side of the axis of the bending moment M cos $(A-\theta)$ looking in the direction of the arrow.

A More Exact Method of Calculating the Bending-Moment Stresses

The preceding method of stress calculation assumes firstly, that the ordinary engineer's theory of bending may be applied to an airscrew blade, and secondly that it is sufficient to regard an axis through the centre of gravity parallel to the chord as the neutral axis of a cross-section. It has been shown by Fage and Collins*—assuming the Engineer's Theory of Bending to be applicable—that the stresses can only be accurately calculated when the position of the neutral axis of each blade section is known.

The method of finding the position of the neutral axis is to resolve the resultant bending moment about the principal axis of the blade section. It has already been shown that the moment of inertia, I_θ, of a blade section about any line passing through the centre of gravity and making an angle θ_1 with the chord may be calculated with good accuracy from the expression, $I_{\theta_1}=0{\cdot}049\sigma^3c^4\cos^2\theta_1+0{\cdot}043\sigma c^4\sin^2\theta_1-0{\cdot}01\sigma^2c^4\sin2\theta_1$. The positions of the principal axes of the section are found from the two values of θ_1, which give the maximum and minimum values of I_{θ_1}. Let OX_0 and OY_0 of Fig. 83 be the principal axes of a blade section, where OX_0 makes an angle Φ with the chord. Also if the angle between the axis of the resultant bending moment and the principal axis OX_0 be denoted by λ, then it is seen that $\lambda=(A-\theta-\Phi)$.

Further,

If I_{x_0}=moment of inertia of the section about the principal axis OX_0,
and I_{y_0}=moment of inertia of section about the principal axis OY_0,
then with the ordinary theory of bending the stress at any point P of the section of which the co-ordinates are x_0 and y_0 will be

$$\left[\frac{M.\cos\lambda.y_0}{I_{x_0}}-\frac{M.\sin\lambda.x_0}{I_{y_0}}\right].$$

The stress at any point on the neutral axis is zero. The equation o the neutral axis is, therefore, $y_0=\left(\frac{I_{x_0}}{I_{y_0}}\right)\tan\lambda.x_0$.

Hence the neutral axis is inclined at an angle δ to the principal axis, where $\tan\delta=\frac{I_{x_0}}{I_{y_0}}\tan\lambda$.

The neutral axis of the section can only be found by resolving along the principal axes. For other axes, the stresses due to the bending moment about one axis will produce a bending moment about the other, because the product of inertia about such axes is not zero. The maximum tensile and compressive stresses of the section occur at those points which are the greatest distances from the neutral axis, such distances being obtained directly by inspection. If y'_c and y'_t are the maximum distances from the neutral axis of points on the periphery of the section, y'_c being measured on the left-hand side of the axis of the bending

* "Some notes on the calculation of the working stresses of an airscrew," by A. Fage, A.R.C.Sc., and H. E. Collins. Advis. Comm. Aeron., 1918.

moment looking in the direction of the arrow (see Fig. 83), then the maximum compressive and tensile stresses due to the resultant bending moment M are

$$\frac{M\cos(\lambda-\delta)y'_c}{(I_{x_0}\cos^2\delta+I_{y_0}\sin^2\delta)} \text{ and } \frac{M\cos(\lambda-\delta)y'_t}{(I_{x_0}\cos^2\delta+I_{y_0}\sin^2\delta)}$$

respectively. The maximum compressive and tensile stresses on the section are, therefore,

$$\left[\frac{M\cos(\lambda-\delta)y'_c}{(I_{x_0}\cos^2\delta+I_{y_0}\sin^2\delta)}-\frac{F}{S_B}\right] \text{ and } \left[\frac{M\cos(\lambda-\delta)y'_t}{(I_{x_0}\cos^2\delta+I_{y_0}\sin^2\delta)}+\frac{F}{S_B}\right]$$

respectively.

With a well-designed modern airscrew the stresses, as calculated from the component of the bending moment about the axis through the C.G. parallel to the chord, are about 20 per cent in error when compared with the more exact method of resolving the bending moment about the neutral axis of the blade section. In view of the extreme difficulty of calculating the *exact* bending-moment stresses it is thought that the former method, which gives some idea of the working stresses and which is supported by practical experience, is sufficiently accurate for nearly all design purposes. In special cases, if it were considered necessary, the stresses could be calculated by the more exact method outlined above.

The principal merit of the first method of stress calculation would appear to be its adaptability to drawing-office practice. It is, however, somewhat bold to assume that the Engineer's Theory of Bending may be applied to such a complex problem ; in fact a direct application of the ordinary theory of bending, if carried to its logical conclusion, rather illustrates the inappropriateness of the method. It has been shown by Fage and Collins* that the shape of the neutral surface of a blade, as calculated from the more exact theory outlined above, is not a plane but a sinuous surface. The curvature of the neutral surface may be such that it may occasionally cut across the fibres of the wood, so that the stresses in such fibres must at such points change from tensile to compressive, an indication of the existence of a somewhat complex state of stress. The problem is further complicated when consideration is taken of the heterogeneity of the physical properties of the material from which the airscrew is made.

The Shear Stresses of an Airscrew Blade

It has been found in practice that an airscrew often fails by cracking longitudinally. It is to be expected that such cracks are caused by shear or transverse bending. In addition, then, to the centrifugal tension and the flexure of the blade about a longitudinal axis there are both torsion and transverse bending about lines more or less parallel to the blade axis, the magnitude of such stresses largely depending on the shape of the blade.

Shearing stresses are of special importance if the airscrew is vibrating considerably under load. With such conditions the fluctuations of the shearing

* Loc. cit.

stresses may result in the heating of the glue and ultimately in the fracture of the airscrew.

The problem of the twisting and stressing of an airscrew blade under load has been investigated both mathematically and experimentally by Griffith and Hague* of the Royal Aircraft Establishment. The method of the mathematical theory is to calculate firstly the position of the line of " principal flexure " of each blade, by applying the ordinary theory of bending to a thin plate of the same shape as the blade when acted on by a load normal to the surface. The blade is assumed to be divided into a number of strips parallel to the direction of the grain of the wood, and the load grading is found which gives the same deflection to all points in any cross-section, each longitudinal strip acting independently of the others. The curve, which passes through the centre of gravity of the load at each section, when drawn on the plan form gives the line of principal flexure, because the loading is such that there is pure bending and no torsion. With any assigned loading the twisting moment is taken as the product of the resultant shear force at the section and the perpendicular from the flexure point on to the line of action of this force. Griffith found that the twist curves of different blades of an airscrew under load may be quite dissimilar because of differences of elastic properties. Experiments have shown that for the principal working parts of two blades of an airscrew the out-of-balance twist may be about six times as great as the tolerances of blade angle allowed in manufacture. With one airscrew the measured twists of two opposite blades, which were built up from the same laminæ, were more or less numerically equal but of opposite sign. Such irregular behaviour can, of course, only be considered when the elastic coefficients of each separate blade are known.

Influence of Shape on the Stresses and Distortions of an Airscrew Blade

Although Griffith's theory of blade deformation may occasionally fail when applied to practice, because of the uncertainty of the magnitude of the elastic coefficients, such a theory may be regarded as a basis from which to determine, for any average values of the elastic coefficients, the influence of shape on the distortion of the blade under load. In nearly all cases it is desirable that the blades of an airscrew should be so shaped as to have a minimum twist under the combined centrifugal and air loadings. Any attempt to design the blades so that they may twist in some definite manner will probably be unsuccessful because of the factors of unknown magnitude which cannot be adequately considered by any theory. The shape of a blade may be conveniently specified in as far as plan and elevation are concerned by reference to the shape of the line joining the centroids of the cross-sections, namely the central line. The elevation view is obtained by projecting on a plane parallel to both the airscrew and blade axes. The plan form is obtained by projecting on a plane at right angles to the airscrew axis. Thus an airscrew blade has a forward tilt when the central line is inclined

* " Preliminary report of the twisting of airscrew blades," by A. A. Griffith, M.ENG., and B. Hague, B.SC. Advis. Comm. Aeron., 1918.

at the tip in the direction of the forward motion ; otherwise a backward tilt. The sweep of the blade is obtained from the plan form, and is known as leading or trailing sweep according as to whether the central line is inclined towards the leading or the trailing edge. Griffith* has shown that with an airscrew blade of small sweep the transverse stresses may be neglected without a great sacrifice of accuracy. It is now proposed to consider in some detail several typical shapes of blade.

Firstly, we shall take the case of a blade which has a forward tilt so that at any section the bending moment due to the centrifugal force acting on the outer part of the blade is balanced, or partially balanced, by the bending moment due to the air loading. The one advantage of this type of blade is that the stresses due to the resultant bending moment are small. For the same maximum stress the blade section may, therefore, be made thinner and so of higher aerodynamic efficiency. Watts† has found from experience, however, that this type of airscrew blade is very weak compared with blades of other shape. In nearly every case of failure under load, cracks developed on the glued joints, although the glue would appear to be quite satisfactory. It was found eventually that a large amount of twisting was set up in airscrews of this type, and that failure was due to the glue working under tension and shear.

Airscrews with which the bending moment due to the air load is more or less balanced by a bending moment due to the centrifugal force may be divided into two groups. Firstly, airscrews with the leading edge approximately straight in both plan and elevation views, and secondly, airscrews with a straight leading edge in elevation and a straight trailing edge in plan form. With both these types of airscrew the blade twists very badly under load and the glue of the joints is under tension. With the latter type the laminæ in plan form are very curved. Griffith‡ and Hague, who have investigated mathematically the twisting of airscrew blades under load, show that it is not possible to eliminate the twisting moment on a blade having a forward tilt—leading edge more or less straight in elevation—by any adjustment of the sweep. They further point out that from the point of view of design a comparatively large twisting moment is a grave disadvantage because of the great difficulty of calculating the torsional stresses.

Another distinctive type of blade shape is that having a more or less straight trailing edge in elevation, that is a backward tilt. With airscrews of this type the bending moment at any section due to the centrifugal force acting on the outer part of the blade is probably large and acts in the same direction as the bending moment due to the air load. An advantage of such a shape is that the glue of the joints is in compression. With either a large trailing or a large leading sweep, e.g. the trailing edge or the leading edge straight in plan form, the blade would tend to twist badly under load.

* Loc. cit.

† "Notes on airscrews with straight trailing and straight leading edges," by H. C. Watts, B.SC. Advis. Comm. Aeron., 1917.

‡ "On the shape of airscrew blades," by A. A. Griffith, M.ENG., and B. Hague, B.SC. Advis. Comm. Aeron., 1918.

To determine how the twisting of an airscrew blade depended on the shape, Watts measured the performances of two airscrews, one with straight leading edge and the other with a straight trailing edge, but identical in all other respects, when driven by the same engine and as nearly as possible at the same time. He found that the rotational speed of the airscrew with the straight trailing edge was about 10 per cent faster than that of the airscrew with the straight leading edge. This difference of rotational speed could only be accounted for by a relative twist of about 2 per cent at the blade tip. To attain consistency it would seem that a designer should obtain from experience the correction factors necessary to make theory agree with practice for any good blade shape, and should then regard such a shape as standard.

Griffith has shown that the adoption of a blade shape which twists so as to give a smaller angle of incidence when climbing than when in horizontal flight, so as to improve the all-round efficiency of the airscrew, is not practicable because of the antagonistic effects of the centrifugal and air forces and also because of the uncertainty of the values of the elastic moduli of the wood. He suggests that a good blade shape may be obtained by giving the blade a slight initial backward tilt so that under the working loads the bending moment due to the centrifugal force is zero, and at the same time such a plan form that there is no twist due to the air load. Another blade shape not quite so good as the preceding is to give the blade a zero sweep so that there is no twist due to the air load and also a slight forward tilt so as to diminish somewhat the bending moment due to the air load. According to Griffith the safe limits of the tensile and shear stresses are reached when the bending moment due to the centrifugal force is about 30 or 40 per cent of the bending moment due to the air load. Although the sections of such a blade may be made thinner, because of the reduction of the resultant bending moment, it is an open question whether the aerodynamic gain which results will compensate for the introduction, by the centrifugal force, of torsional stresses. It is pointed out by Griffith and Hague that wherever possible the blade laminations should be straight and as straight-grained as possible. When abnormal laminations are used they should be placed at the middle of the blade, since the stresses of the middle laminæ which contain the flexual centres are small.

Although longitudinal cracks have very little effect on the strength of laminæ which are subjected to pure end-grain stresses, it is almost certain with a blade having a large trailing sweep that the starting of a longitudinal crack must be followed by the complete separation of the trailing edge. On the other hand, with a small sweep it is probable that nothing more serious than a loss of power would result from such a crack.

It has been shown by Watts that the stresses of the bending moment due to the centrifugal force may be greatly reduced by varying the position of the boss relative to the blade. A sketch of this form of blade, which has been largely adopted at the Air Ministry, is shown in Fig. 96. It will be seen from the plan form that the trailing edge is practically a straight line passing through the centre of the airscrew boss. The trailing edge is also straight in elevation. The centres of gravity of each section are marked in both the plan form and the elevation.

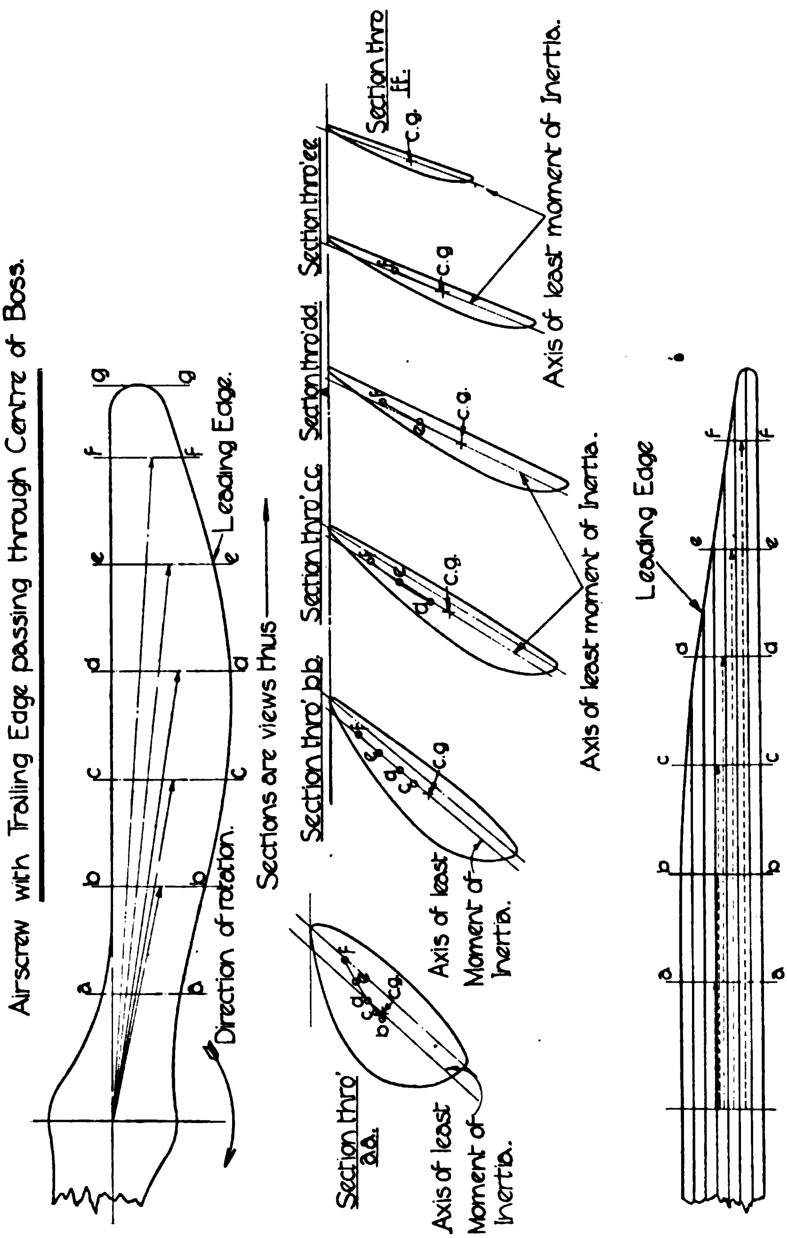

Airscrew with Trailing Edge passing through Centre of Boss.
Direction of rotation.
Leading Edge.
Sections are views thus
Section thro' aa.
Axis of least Moment of Inertia.
Section thro' bb.
Section thro' cc.
Section thro' dd.
Section thro' ee.
Section thro ff.
c.g.
Axis of least moment of Inertia.
Leading Edge

FIG. 96.

The centrifugal force on any section acts along the line joining the centre of gravity of the section to the centre of the boss, so that the points f, e, d, c, b in the sectional views show where the lines of the centrifugal pull cut through the sectional planes. The bending moment at a section due to the centrifugal force on the outer part of the blade can now be easily calculated. It will be noticed from Fig. 96 that, at any section, the lines of action of the centrifugal pulls of the outer sections practically intersect the axis passing through the centre of gravity of the section parallel to the chord, so that the bending moment about this axis due to the centrifugal force is very small. There is, of course, a bending moment about the axis through the centre of gravity of the section at right angles to the chord, but such a bending moment is of small importance because the moment of inertia of the section about this axis is large. Watts suggests that to minimise the local shear along the grain of the wood at the boss care should be taken that the blade does not run too sharply into the boss at the trailing edge. With this method of design it should be noticed that the glue of the joints is not working in tension.

CHAPTER X

ON THE VIBRATIONS OF AN AIRSCREW

(a) The Gyroscopic Vibrations of an Airscrew

Mathematical Theory.—The subject of the gyroscopic vibration of an airscrew has been considered mathematically by Jones* in a paper published by the Advisory Committee for Aeronautics. It is now proposed to investigate, in a manner suggested by this paper, the vibratory motion due to gyroscopic action of the airscrew when an aeroplane is making a right-handed turn with a uniform angular velocity Ω. The rectangular axes taken are as shown in Fig. 97. The airscrew has a uniform angular velocity ω about the axis OX, which coincides with the airscrew axis. The airscrew rotates, therefore, in the plane YOZ, in a right-handed direction when looking in the positive direction of OX. The axis OZ

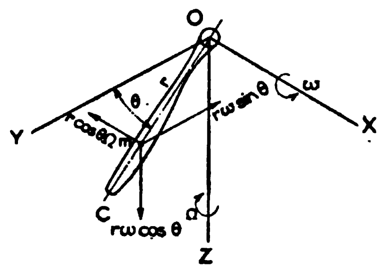
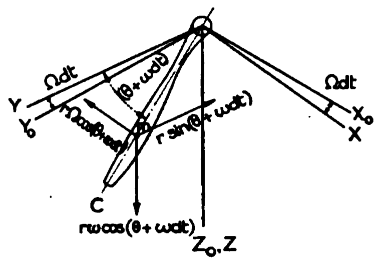


Fig. 97.



Airscrew blade in the plane YOZ.

Fig. 98.

may be regarded as more or less parallel to the normal axis of the aeroplane. This system of rectangular axes OX, OY, and OZ is fixed in the aeroplane. At any time, t, the position of an airscrew blade is as shown by the line OC of the diagram of Fig. 97. At a time, (t+dt), the positions of the axes OX, OY, and OZ relative to the positions at time t are as shown in the diagram of Fig. 98. It will be noticed that the axes OX and OY have moved through an angular distance Ωdt, and that the blade axis OC now makes an angle $(\theta+\omega dt)$ with the axis OY. Consider an element of mass, m, on the blade axis at a radial distance, r, from the axis of rotation. At any time, t, the velocities parallel to the axes OX, OY, and OZ of this mass will be $-r\Omega\cos\theta$, $-r\omega\sin\theta$, and $r\omega\cos\theta$ respectively. At a time, (t+dt), the velocities will be $-r\Omega\cos(\theta+\omega dt)$, $-r\omega\sin(\theta+\omega dt)$, and $r\omega\cos(\theta+\omega dt)$ respectively.

* "Gyroscopic action and propeller vibration." Summary of notes by Mr. F. W. Lanchester and Mr. B. Melville Jones. Advis. Comm. Aeron., 1914–15.

Hence the change of velocity in the direction OX_0

$$=[-r\Omega\cos(\theta+\omega dt)+r\omega\Omega dt\sin(\theta+\omega dt)+r\Omega\cos\theta]=2r\Omega\omega\sin\theta\, dt.$$

Hence the acceleration $=2r.\Omega.\omega.\sin\theta$.

There is therefore a force $2r\frac{m}{g}\Omega.\omega\sin\theta$, acting on the mass m in a direction perpendicular to the plane YOZ, so that the reaction couple about the axis OY, which is exerted by the blade on the aeroplane,

$$=-\int\frac{2m}{g}r^2\Omega\omega\sin^2\theta=\frac{\omega\Omega}{g}\cdot I_B\cdot[\cos 2\theta-1],$$

where I_B =moment of inertia of the blade about the axis of rotation.

It will be noticed that the moment of inertia of the blade about the blade axis is neglected.

Also the reaction couple about OZ exerted by the blade

$$=+\int 2r^2\Omega\omega\sin\theta\frac{m}{g}\cos\theta=+\frac{\omega.\Omega.I_B}{g}\sin 2\theta.$$

The couple about the axis OY has a mean value of $-\frac{I_B\omega\Omega}{g}$ and varies from 0 to $-\frac{2\omega\Omega I_B}{g}$ twice per revolution. The couple about the axis OZ varies from $-\frac{I_B\omega\Omega}{g}$ to $\frac{I_B\omega\Omega}{g}$ twice in each revolution and has a mean value of zero.

We see, then, that when the airscrew turns with an angular velocity Ω about the axis OZ there are gyroscopic couples $-\frac{\omega\Omega I_B}{g}$ and $\frac{\omega\Omega I_B\cos 2\theta}{g}$ about the axis OY, and $-\frac{\omega I_B\Omega\sin 2\theta}{g}$ about OZ. The latter two couples are equivalent to a steady couple $\frac{\omega\Omega I_B}{g}$ about an axis at right angles to the axis of the airscrew, but which revolves in the plane of the airscrew with angular velocity 2ω.

Hence each blade of an airscrew contributes an average couple of $-\frac{I_B\omega\Omega}{g}$ about an axis at right angles to both the axis of turning of the aeroplane and the airscrew axis, together with a couple $\frac{I_B\omega\Omega}{g}$ about an axis which is at right angles to the axis of, and rotates with, the airscrew, but at twice the angular velocity, and also a force $\frac{2\omega\Omega}{g}\Sigma mr\sin\theta$ perpendicular to the plane of the airscrew.

With a two-bladed airscrew the blades are set at an angle π with one another. The rotating axes coincide therefore, the resultant couple about them being $\frac{2I_B\omega\Omega}{g}$, that is the mean gyroscopic couple of the airscrew.

With a three-bladed airscrew the blades are set at $\left(\frac{2\pi}{3}\right)$, so that the angle between the corresponding rotating axes are $\left(\frac{4\pi}{3}\right)$ and the couples are therefore in equilibrium. Similarly, with an airscrew of n blades spaced $\left(\frac{2\pi}{n}\right)$ apart the corresponding revolving axes are spaced $\left(\frac{4\pi}{n}\right)$ apart, so that if n be greater than 2 the resultant revolving couple will be zero. The varying force of magnitude $\frac{2\omega\Omega}{g}\ \Sigma mr \sin\theta$ has a zero value, if the centre of gravity of the airscrew is in the axis of rotation. Two general conclusions may therefore be drawn from the investigation.

(*a*) An airscrew with more than two equally spaced blades and with the centre of gravity on the axis of rotation, will have a steady gyroscopic couple when the aeroplane is turning.

(*b*) A two-bladed airscrew gives a steady gyroscopic couple plus a couple of the same magnitude as this gyroscopic couple, acting about an axis which rotates in the plane of the airscrew, with twice the speed of, and in the same direction as, the airscrew.

It has been pointed out by Lanchester* that with a two-bladed airscrew there is a variation of the moment of inertia about a vertical axis from a minimum value which is approximately zero to a maximum value; this variation occurs twice a revolution. It follows, then, that a uniform turning motion of the aeroplane needs an applied turning moment of variable magnitude. This, then, would be another cause for vibratory motion with a two-bladed airscrew since in practice the applied steering torque would not fluctuate in harmony with the periodic change of the moment of inertia of the aeroplane.

We see, then, that the pitching or yawing of an aeroplane driven by a four-bladed airscrew induces a gyroscopic couple of steady magnitude. The magnitude of the gyroscopic couple is equal to the product of the angular momentum of the rotating airscrew and the angular speed of pitching or yawing as the case may be. Moreover, the direction of the induced gyroscopic couple is such that the aeroplane tends to turn in such a manner that the direction of rotation of the airscrew is the same as that of the extraneous couple. With a two-bladed airscrew, in addition to a uniform gyroscopic couple when the aeroplane is yawing or pitching, there is also a couple which acts about an axis which rotates in the plane of the airscrew and so introduces a vibratory motion.

Application of the Preceding Theory.—Suppose the rudder is put over so that the applied air couple tends to turn, to the right, an aeroplane which is driven by a right-handed tractor—that is the tractor has a clockwise direction when viewed from the pilot's seat. Then in this case the aeroplane will tend to dive. Further, to take another illustration, if we assume that the applied air couple acts in the

* Loc. cit.

direction which would make the aeroplane dive, the gyroscopic action would be such that a horizontal left-handed turn would be made.

The method of calculation may perhaps be made clear by a practical case.

Imagine an aeroplane on which is mounted a rotary engine to be turning to the right with an angular velocity of $2\pi/10$ radians per sec., and assume—

The weight of the rotary engine	$=300$ lb.
The weight of the four-bladed airscrew	$=35$ lb.
The radius of gyration of the engine	$=1{\cdot}0$ ft.
The radius of gyration of the airscrew	$=2{\cdot}5$ ft.
Rotation speed of engine and airscrew	$=1200$ r.p.m.

Then,

$$\text{Angular momentum of engine} = \left[\frac{300 \times 1{\cdot}0 \times 40\pi}{32{\cdot}2}\right]$$

$$= 373\pi \text{ (ft. lb. sec. units)}$$

$$\text{Angular momentum of airscrew} = \left[\frac{35 \times 6{\cdot}25 \times 40\pi}{32{\cdot}2}\right]$$

$$= 272\pi \text{ (ft. lb. sec. units)}$$

$$\text{Angular velocity of turning} = \pi/5 \text{ radians per sec.}$$

Hence the gyroscopic pitching moment acting on the machine due to the angular velocity of turning $=(373\pi+272\pi)\pi/5=1270$ lb.-ft.

During a manœuvre the motion of an aeroplane depends on both the extraneous air couples and the induced gyroscopic couples, and so cannot be calculated in as simple manner as shown above.

(b) The Whirling and Transverse Vibrations of a Rotating Airscrew and its Shaft

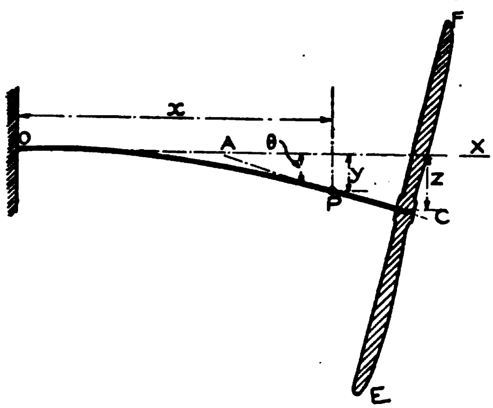
Mathematical Theory.—The whirling speed of a modern airscrew as now mounted on the engine of an aeroplane is very high and much greater than the bursting speed of the airscrew blade. It may be occasionally desirable, however, to rotate an airscrew at the end of a long shaft, in which case it is necessary to calculate the whirling speed of the combination of airscrew and shaft. The general mathematical theory of the transverse vibrations of rotating systems has been adequately treated by several writers of whom mention should perhaps be made of Dunkerley, Chree, and Greenhill. Investigations of the particular problem of the transverse vibrations of a rotating airscrew and its shaft have been made by Fage* and later by Berry.† It is perhaps desirable at the outset to indicate the relationship between the frequency of the lateral transverse vibrations and the whirling speed.

* "The whirling and transverse vibrations of a rotating airscrew and its shaft," by A. Fage, A.R.C.Sc. Advis. Comm. Aeron., 1917.

† "Report on the whirling of an airscrew shaft," by A. Berry. Advis. Comm. Aeron., 1918.

Taking the simple case of a shaft of negligible mass fixed at one end and carrying a mass M at the other, it can easily be shown, if $\frac{p}{2\pi}$ be the frequency of the transverse vibrations of the mass M when the shaft is not rotating, and $\frac{q}{2\pi}$ the frequency of the transverse vibrations when the shaft is rotating with an angular velocity ω, that $q^2 = p^2 - \omega^2$.

The elastic forces of the shaft tend to restore the mass to its equilibrium position, but the centrifugal force acting on the mass has the opposite tendency, and by reducing the magnitude of the righting force diminishes also the frequency of vibration of the rotating mass and shaft. If ω be equal to p, the frequency of the transverse vibrations is zero, so that the mass is in neutral equilibrium no matter what the value of the lateral displacement may be, and hence if the lateral displacement be sufficiently great the shaft will fracture. A shaft in such a state is said to "whirl," the righting force due to the elasticity of the system having been reduced to zero by the centrifugal force called into being by deflection. For this particular instance the frequency of rotation when the shaft is whirling, and the frequency of the transverse vibrations when the shaft is not rotating, have equal values, but this is not the case with a rotating airscrew and shaft, in which case moments of inertia and damping need consideration.



Diagrammatic Sketch of Vibrating Airscrew.
Plane of vibration coincident with plane of paper.
EF - Axis of Airscrew Blades.

FIG. 99.

It is now proposed to describe in outline only, the method adopted by the author* to calculate the frequency of the transverse vibrations of a rotating airscrew and shaft. The investigation is an application of Lagrange's equations for small oscillations.

An instantaneous position of the rotating and vibrating airscrew and shaft is shown in Fig. 99, the plane of vibration of the shaft being coincident with the plane of the paper. It is assumed that the airscrew shaft is fixed in direction at the end O, so that the line OX represents the mean position of the vibrating shaft, and also the axis about which the rotation is taking place.

The following symbols are used in the investigation :—

L = length of the shaft.
M = mass of the airscrew.
m = mass of the shaft per ft. run.
I = moment of inertia of the airscrew about the axis CA.

* Loc. cit.

I_1 = moment of inertia of the airscrew about an axis through C at right angles to the plane of bending.

I_2 = moment of inertia of the airscrew about an axis through C in the plane of bending and at right angles to CA.

ω = speed of rotation of the airscrew.

E = Young's modulus of elasticity.

I_B = moment of inertia of a cross-section of the shaft about a diameter.

x = distance from O, of any point P on the axis of the shaft.

y = deflection of the point P from the line OX.

At any time t, the free end of the shaft is displaced a distance " z " from its mean position, and the plane of the airscrew has an angular displacement " θ " from its mean position.

The equations of motion are expressed in terms of the variables θ and z.

We have when $x=0$, $\frac{dy}{dx}=0$ and $y=0$,

and $x=L$, $\frac{dy}{dx}=\theta$ and $y=z$.

So that the equation of the curve of the centre line of the shaft is

$$y=\left(\frac{3z-\theta L}{L^2}\right)x^2+\left(\frac{\theta L-2z}{L^3}\right)x^3.$$

Lagrange's equations for a damped vibrating system in which there are two variables θ and z are

$$\frac{\delta}{\delta t}\frac{(\delta T)}{(\delta\dot{\theta})}-\frac{\delta T}{\delta\theta}+\frac{\delta V}{\delta\theta}+\frac{\delta F}{\delta\dot{\theta}}=0$$

and
$$\frac{\delta}{\delta t}\frac{(\delta T)}{(\delta\dot{z})}-\frac{\delta T}{\delta z}+\frac{\delta V}{\delta z}+\frac{\delta F}{\delta\dot{z}}=0$$

where T = kinetic energy of the whole system.

V = potential energy of the whole system.

and F = half the rate at which energy is being dissipated.

The kinetic energy of the whole system

$$=\left(\int_0^L\frac{1}{2}mdx[\dot{y}^2+(y\omega)^2]^2\right)+\frac{M}{2}(\dot{z}^2+\omega^2z^2)+\frac{I\omega^2}{2}\cos^2\theta+\frac{I_2\omega^2}{2}\sin^2\theta+\frac{I_1{}^2}{2}$$

$$=\left\{\begin{array}{l}\frac{m\omega^2L}{2}\left[\frac{1}{5}(3z-\theta L)^2+\frac{1}{3}(3z-\theta L)(\theta L-2z)+\frac{1}{7}(\theta L-2z)^2\right]\\ +\frac{mL}{2}\left[\frac{1}{5}(3\dot{z}-\dot{\theta}L)^2+\frac{1}{3}(3\dot{\theta}-\dot{\theta}L)(\dot{\theta}L-2\dot{z})+\frac{1}{7}(\dot{\theta}L-2\dot{z})^2\right]\\ +\frac{M}{2}(\dot{z}^2+\omega^2z^2)+\frac{I\omega^2}{2}+\frac{I_1}{2}(\dot{\theta}^2-\omega^2\theta^2)\end{array}\right\}$$

since $I=I_1+I_2$ and $\sin\theta=\theta$ nearly.

The potential energy, V, of the system is equal to the potential energy of the shaft.

$$\text{Hence } V=\int_0^L \frac{EI_B}{2}\left(\frac{d^2y}{dx^2}\right)^2 dx$$
$$=\int_0^L \frac{EI_B}{2}\left[\frac{2}{L^2}(3z-\theta L)+\frac{6x}{L^3}(\theta L-2z)\right]^2 dx$$
$$=\frac{EI_B}{L^3}[3z^2-3z\theta L+\theta^2L^2].$$

Owing to the vibratory motion of the airscrew blades through the air, energy is dissipated, the rate of such dissipation—2F—being equal to $K\dot{\theta}^2$, where K is a constant. The value of the damping factor " K " can be readily calculated when the aerodynamic data of the blade sections, and also the working conditions of the airscrew are known.

Substituting the above values of T, V, and F in Lagrange's equations, and collecting the terms in a convenient form, we get

$$\left\{\begin{array}{c}\left[\left(I_1+\frac{2}{210}mL^3\right)D^2+KD+\left(I_1\omega^2-\frac{2}{210}m\omega^2L^3+\frac{4EI_B}{L}\right)\right]\theta \\ -\left[\frac{11}{210}mL^2D^2+\left(\frac{6EI_B}{L^2}-\frac{11}{210}m\omega^2L^2\right)\right]z\end{array}\right\}=0$$

and

$$\left\{\begin{array}{c}-\left[\frac{11}{210}mL^2D^2+\left(\frac{6EI_B}{L^2}-\frac{11}{210}m\omega^2L^2\right)\right]\theta \\ +\left[\left(M+\frac{78}{210}mL\right)D^2+\left(\frac{12EI_B}{L^3}-M\omega^2-\frac{78}{210}m\omega^2L\right)\right]z\end{array}\right\}=0,$$

so that the complete motion is given by the solution of the differential equation

$$\left\{\begin{array}{c}\left[\left(I_1+\frac{2}{210}mL^3\right)D^2+KD+\left(I_1\omega^2-\frac{2}{210}m\omega^2L+\frac{4EI_B}{L}\right)\right]\left[\left(M+\frac{78}{210}mL\right)D^2+\frac{12EI_B}{L^3}-M\omega^2-\frac{78}{210}m\omega^2L\right)\Big] \\ -\left[\frac{11}{210}mL^2D^2+\left(\frac{6EI_B}{L^2}-\frac{11}{210}m\omega^2L^2\right)\right]^2\end{array}\right\}=0$$

The solution of the above equation in its present form would be somewhat tedious. Accordingly, a systematic series of calculations was made by the author to ascertain how the frequency of the vibrations depended on the damping, the mass and length of the shaft, and the mass and moment of inertia of the airscrew.

The conclusions made from the results of these calculations are :—

(*a*) The air damping of an airscrew is negligible, in so far as the frequency of the transverse vibrations is concerned.

(*b*) The whirling speed may be calculated with very good accuracy by assuming the effect of the shaft to be the equivalent of a mass $\left(\frac{78}{210}mL\right)$ at its end.

In this case the whirling speed is overestimated, the inaccuracy increasing with the length of the shaft.

(*c*) As would be expected, neglecting the mass of the shaft makes the whirling speed too high, the discrepancy increasing with the length of the shaft.

(*d*) Even when the moment of inertia I_1 of the airscrew is small it should not be neglected in the calculation of the whirling speed.

(*e*) The critical speed of an airscrew and shaft increases with the moment of inertia of the airscrew.

(*f*) When the moments of inertia of the airscrew and of the shaft are neglected the whirling speed has the same frequency as the natural period of the transverse vibration of the airscrew when not rotating.

Neglecting the air damping of the airscrew blades and assuming that the effect of the inertia of the shaft is equivalent for all practical purposes to a mass $\left(\frac{78}{210}mL\right)$ placed at its end, the differential equation of the motion of the airscrew and shaft then becomes

$$\left\{\left[D^2+\omega^2+\frac{4EI_B}{LI_1}\right]\left[D^2-\omega^2+\frac{12EI_B}{L^3\left(M+\frac{78}{210}mL\right)}\right]-\left(\frac{36E^2I_B^2}{I_1L^4\left(M+\frac{78}{210}mL\right)}\right)\right\}\theta=0$$

that is

$$\left\{D^4+D^2\left(\frac{4EI_B}{LI_1}+\frac{12EI_B}{L^3\left(M+\frac{78}{210}mL\right)}\right)+\left(\omega^2+\frac{4EI_B}{LI_1}\right)\left(\frac{12EI_B}{L^3\left(M+\frac{78}{210}mL\right)}-\omega^2\right)\right.$$
$$\left.-\frac{36E^2I_B^2}{I_1L^4\left(M+\frac{78}{210}mL\right)}\right\}\theta=0.$$

The whirling speed, that is the rotational speed at which the frequency of one of the transverse vibrations becomes zero, is then calculated from the expression

$$\left[\omega^2-\frac{12EI_B}{L^3\left(M+\frac{78}{210}mL\right)}\right]\left[\omega^2+\frac{4EI_B}{LI_1}\right]+\frac{36E^2I_B^2}{I_1L^4\left(M+\frac{78}{210}mL\right)}=0,$$

which may be written

$$\omega^4+\omega^2\left[\frac{4EI_B}{LI_1}-\frac{12EI_B}{L^3\left(M+\frac{78}{210}mL\right)}\right]=\frac{12E^2I_B^2}{I_1L^4\left(M+\frac{78}{210}mL\right)}.$$

If the above equation be solved for ω^2 it will be found that there is one positive and one negative value.

The whirling speed may be got by taking the square root of the positive value of ω^2.

Formula for the Calculation of the Whirling Speed of an Airscrew and Shaft. The whirling speed, ω, of a rotating airscrew and shaft, the shaft being a cantilever

fixed in direction at one end, is obtained, then, from the square root of the positive value of ω^2 as calculated from the solution of the equation,

$$\omega^4+\omega^2\left(\frac{4EI_B}{LI_1}-\frac{12EI_B}{L^3\left(M+\frac{78}{210}mL\right)}\right)=\frac{12E^2I_B^2}{I_1L^4\left(M+\frac{78}{210}mL\right)},$$

where ω = rotational speed in radians per sec.

M = mass of the airscrew in slugs.

I_1 = moment of inertia of the airscrew about an axis passing through the centre of the airscrew at right angles to the plane of the transverse vibrations (slug-ft.^{2}).

L = length of the shaft in feet.

m = mass of one foot length of shaft in slugs.

E = Young's modulus in lb. per square foot.

I_B = moment of inertia of a cross-section of the shaft about a diameter in (feet)^4.

If $I_1=0$ the whirling speed may be calculated from the equation

$$\omega^2=\frac{3EI_B}{L^3\left(M+\frac{78}{210}mL\right)}.$$

(c) THE TRANSVERSE VIBRATIONS OF AN AIRSCREW BLADE

Mathematical Theory.—The design of an airscrew cannot be regarded as satisfactory unless an attempt is made to calculate or to measure experimentally, the frequency of the transverse vibrations of the blade. Needless to say, if this natural frequency has the same magnitude as a rotational speed of practice, forced vibrations are induced which may ultimately result in the fracture of he blade.

This subject has been investigated, both theoretically and experimentally, by A. A. Griffith* of the Royal Aircraft Establishment. This mathematical theory will now be considered in outline.

In as far as the calculation of the frequency of the transverse vibrations is concerned, an airscrew blade may be regarded as an elastic cantilever of arbitrary cross-section, so that the vibrations include torsion as well as flexure. With a blade of well-designed shape, however, the torsion of the blade may be neglected, so that the general equation of the transverse vibrations is

$$\frac{\delta^2}{\delta x^2}\left(EI_c\frac{\delta^2 y}{\delta x^2}\right)+\frac{\Delta S_B}{g}\frac{\delta^2 y}{\delta t^2}=0,$$

where E = Young's modulus.

g = acceleration due to gravity.

Δ = density of the material.

f = frequency of vibration.

* "A formula for calculating the vibration speeds of airscrews," by A. A. Griffith, M.ENG. Advis. Comm. Aeron., 1918.

S_B = area of blade section.

I_c = moment of inertia of a blade section about an axis through C.G. parallel to the chord.

t = time.

x = distance of the section from the axis of the airscrew.

y = deflection at x at time t.

y_0 = amplitude at x.

Writing $x=\alpha R$ where R = length of the blade, and assuming a simple harmonic motion of the type $y=y_0 \sin 2\pi.f.t.$ for the fundamental mode, it follows from the general equation, that

$$\frac{d^2}{dx^2}\left(I_c\frac{d^2y}{dx^2}\right)=\frac{\Delta S_B}{E.g}4\pi^2f^2y_0$$

and $\therefore\ y_0=\dfrac{4\pi^2 f^2 \Delta R^4}{E.g}\displaystyle\int_0^\alpha\int_0^\alpha\frac{1}{I_c}\int_1^\alpha\int_1^\alpha S_B y_0.d\alpha.d\alpha.d\alpha.d\alpha.$

Writing $y_0=\dfrac{1}{K_S}\displaystyle\int_0^\alpha\int_0^\alpha\frac{1}{I_c}\int_1^\alpha\int_1^\alpha S_B y_0.d\alpha.d\alpha.d\alpha.d\alpha$

it follows that $f=\dfrac{1}{2\pi R^2}\sqrt{\dfrac{Eg}{\Delta K_S}}$.

Hence it is necessary to evaluate K_S in order to determine f.

In order to establish the general formula given below Griffith assumed that $y_0=y_1\alpha^3$, y_1 being the deflection at the free end. Use was made of the expressions

$$S_B=A+B\alpha+C\alpha^2+D\alpha^3+F\alpha^4$$

$$\text{and}\quad \frac{1}{I_B}=P+Q\alpha^3+R\alpha^6+S\alpha^9+T\alpha^{12},$$

the values of the constants being determined by substituting the specified moments of inertia and areas at the points $\alpha=0{\cdot}1$, $0{\cdot}3$, $0{\cdot}5$, $0{\cdot}7$, and $0{\cdot}9$.

Griffith's Rule for the Calculation of the Fundamental Vibration Speed.—Find the sectional areas S_{B1}, S_{B2}, S_{B3} S_{B4}, S_{B5} and the moments of inertia I_{c1}, I_{c2}, I_{c3}, I_{c4}, and I_{c5} at points distance 0·1R, 0·3R, 0·5R, 0·7R, and 0·9R respectively from the centre of the boss, R being the radius of the airscrew. Reference is then made to the following table of coefficients.

TABLE XVII

Sectional areas.	Moments of inertia.				
	I_{c1}	I_{c2}	I_{c3}	I_{c4}	I_{c5}
$+S_{B1}$	16·75	10·91	3·285	1·305	0·070
$-S_{B2}$	18·68	52·11	17·16	6·69	0·363
$+S_{B3}$	207·50	120·60	35·82	13·99	0·777
$-S_{B4}$	82·73	80·58	22·29	15·01	0·886
$+S_{B5}$	392·30	221·00	74·21	22·98	0·886

Divide each of the numbers in the above table by the moment of inertia shown at the head of the column and multiply by the area shown at the beginning of the row.

Find the sum of the quantities so calculated, remembering that those containing S_{B2} and S_{B4} are negative. Call the sum K_S.

Then Frequency per minute $=\dfrac{18{,}770}{R^2}\sqrt{\dfrac{E}{\Delta K_S}}$,

where E is Young's modulus in lb./sq. in.
Δ is the density of the material in lb./cu. in.
R is the radius in inches
and the areas and moments of inertia are in inch units.

The above formula is accurate for blades having a more or less symmetrical plan form. Where the asymmetry of the plan form is such that the trailing edge is straight the calculated frequency may be about 20 per cent too high. It is probable that the second speed of vibration may be about three times the fundamental.

A Practical Application of Griffith's Rule.—For the purpose of illustration the frequency of vibration of a blade of the airscrew, of which a sketch is given in Fig. 84, is now calculated.

With this airscrew the values of S_{B1}, S_{B2}, S_{B3}, S_{B4}, and S_{B5} are 5·0, 5·0, 4·7, 3·75, and 1·8 respectively. Also the values of I_{c1}, I_{c2}, I_{c3}, I_{c4}, and I_{c5} are 0·75, 0·46, 0·275, 0·125, and 0·025 respectively. Hence $K_S=3125$. The frequency per minute $=\dfrac{18{,}770}{(54)^2}\sqrt{\dfrac{1{\cdot}7\times10^6}{{\cdot}0243\times3125}}=965{\cdot}0$

since $R=54$ in.
$\Delta=0{\cdot}0243$ lb. per cub. in.
and $E=1{\cdot}7\times10^6$ lb. per sq. in.

CHAPTER XI

SOME SPECIAL AIRSCREW PROBLEMS

(*a*) Dependence of Efficiency on those Airscrew Parameters which are Influenced by Engine Gearing

It is now proposed to consider in some detail how the efficiency of an airscrew, as given by the expression $\frac{1}{(1+a)} \cdot \frac{\tan\psi}{\tan(\psi+\gamma)}$, depends on the principal parameters of design, such as speed of the aeroplane, the diameter and rotational speed of the airscrew; and the horse-power of the engine. It has been shown that the inflow factor $\frac{1}{(1+a)}$ is the efficiency as calculated from the momentum theory of Froude, in which both the frictional losses and the rotational motions of the air —regular and irregular—are neglected. Expressed briefly, the Froude régime considers only the axial translation motions necessary for the creation of the thrust. The value of the factor $\frac{\tan\psi}{\tan(\psi+\gamma)}$, which depends on the shape of a blade section, the angle of incidence, and the pitch angle ψ, is, however, calculated from aerofoil data measured experimentally in a wind channel, and so takes account in some measure of both the frictional losses at the airscrew disc and the rotational motions, etc., of the actual régime. From calculations of the performances of a large number of modern airscrews the author has found that the average value of $\frac{\tan\psi}{\tan(\psi+\gamma)}$, at the working conditions of practice, ranges from about 0·80 to 0·85. With the particular purpose of showing how the factor $\frac{1}{(1+a)}$ of the efficiency expression depends on the diameter, the forward speed, and the thrust of the airscrew, it will be assumed firstly that $\frac{\tan\psi}{\tan(\psi+\gamma)}$ has a constant value of 0·82, and secondly that the relationship between T, V, D, and "a," is given by the Froude expression $T=\frac{\pi D^2}{4}\rho(1+a)2V^2a$. These assumptions do not appreciably affect the accuracy of the general conclusions. With such assumptions we have, then, $(T/\rho D^2V^2)=\frac{129}{\eta}\left(\frac{82}{\eta}-1\right)$, where η is the percentage efficiency.

From this expression it is seen that with constant values of the thrust and forward speed the efficiency increases with the diameter, for the obvious reason that the larger the area of the airscrew disc the smaller will be the inflow velocity and so the greater the efficiency. The performance curve of any one airscrew mounted on an aeroplane with the engine working with a constant setting of the throttle controls, shows that the thrust increases with a decrease of the forward speed. It is both this decrease of forward speed and increase of thrust which partly accounts for the decrease of efficiency at the climb. It is therefore important to realise that even if an airscrew be designed to have a high value of $\frac{\tan \psi}{\tan (\psi+\gamma)}$ at the climb—this may be obtained by using blade sections of a good aerofoil shape working at the best angle of incidence—then the value of the efficiency will be small, if the thrust be large, the diameter of the airscrew small, and the forward speed at the climb also small. The data of Table XVIII

TABLE XVIII

Remarks.	Diameter, ft.	Forward speed, m.p.h.	Value of T lb.	Value of $\left(\frac{1}{1+a}\right)$	Probable efficiency percentage.
Representative working conditions for an airscrew of an aeroplane of moderate size.	8 ft. 6 in.	125 (maximum)	325	0·965	79·0
		85 (climbing)	450	0·910	74·5
Representative working conditions for an airscrew of a large aeroplane driven by several airscrews.	12 ft. 0 in.	100 (maximum)	650	0·950	78·0
		70 (climbing)	900	0·875	72·0

are more or less average values of $\left(\frac{1}{1+a}\right)$ as calculated for representative airscrews working at the maximum horizontal flight speed and also at the climb. From a reference to this table it will be seen that this inflow factor varies from about 0·96 at the maximum horizontal flight speed to about 0·90 at the climb.

It has been found at the National Physical Laboratory* that when designing an airscrew to develop a thrust of 550 lb. at a forward speed of 100 m.p.h. there are quite large ranges of rotational speed and of diameter over which the factor $\left(\frac{\tan \psi}{\tan (\psi+\gamma)}\right)$ has a good efficiency. We also found that a two-bladed airscrew of large diameter will need to rotate at a low speed, say at 900 r.p.m., when it is mounted on an engine of low power, in order to make use of good blade sections.

* "The relation between the efficiency of a propeller and the speed of rotation," by L. Bairstow, A.R.C.SC., A. Fage, A.R.C.SC., and H. E. Collins. Advis. Comm. Aeron., 1916.

With an increase of engine power a change may be made to four blades, but at quite moderate powers the blades become very wide at a low rotational speed if the maximum diameter be limited to about 9 ft. There is in consequence a sacrifice of efficiency. On the other hand, with a fast-running airscrew on an engine of low power, either the diameter becomes excessively small with a corresponding fall of efficiency or the blades become very long and narrow. In the latter case the diameter is limited by the maximum stress which the material of the airscrew can withstand. We also found that a good efficiency was not easily obtainable on an airscrew of a diameter of 9 ft. rotating at about 1600 r.p.m. at a forward speed of 100 m.p.h. until the power absorbed was greater than 250 h.p.; for powers less than 250 h.p. the design of the blade is such that a low value of the factor $\frac{\tan \psi}{\tan (\psi+\gamma)}$ appears to be inevitable, chiefly because each blade section is working at a smaller angle of incidence than that corresponding to the maximum aerodynamic efficiency of the blade section. With these values of the rotational speed and horse-power a two-bladed airscrew appears to be the better, and more and more power can be absorbed, without sacrificing either strength or efficiency, firstly by increasing the blade widths of a two-bladed airscrew, and when these become sufficiently great by changing to four blades.

The preceding discussion of the dependence of the efficiency of an airscrew on the speed of rotation and on the diameter has a direct bearing on the desirability of introducing another parameter in the design of an airscrew, namely a reduction gearing between the engine and the airscrew. With an engine which develops a large horse-power at a high rotational speed it may be necessary, in order to obtain the best over-all performance of an aeroplane, to run the airscrew at a lower rotational speed than that of the engine. In view of the additional mechanical complication, the employment of engine gearing is only justifiable when it favours a better over-all efficiency of the aeroplane, consideration being taken of the additional weight of, and the energy dissipated in, the gearing. With a high-speed airscrew the diameter should not be too large, otherwise the speed at the tip will approach too closely the velocity of sound. Thus, assuming the tip speed to be limited to 850 ft. per sec. the maximum diameters of airscrews rotating at 1800 r.p.m. and 1600 r.p.m. are 9 ft. and 10 ft. respectively.

In view of the large number of variables involved in the design of an airscrew it is a somewhat difficult matter to treat generally the subject of engine gearing. A general treatment is only applicable where the ranges of the horse-power, the rotational speed of the engine, and the forward speed of the aeroplane are definitely specified. An investigation of a somewhat particular nature was made at the National Physical Laboratory* to show how the ratio of the reduction gearing modified the design and performance of an airscrew when mounted on an engine which developed 300 h.p. at 1800 r.p.m. at an altitude of 10,000 ft., the forward speed of the aeroplane being 125 m.p.h. The procedure adopted was to design

* "Dependence of the efficiency of an airscrew on the speed of rotation and the diameter, with a direct reference to the question of engine gearing," by A. Fage, A.R.C.SC., and H. E. Collins. Advis. Comm. Aeron., 1918.

a series of airscrews to absorb the same horse-power at the same forward speed, the rotational speed of each airscrew depending on an assumed ratio of the reduction gearing of the engine. Subject only to the limitations of the maximum stress allowable in the material of the airscrew blades and also to the maximum tip speed, endeavour was made to design in each case the most efficient airscrew for the fixed parameters by suitably adjusting the remaining characteristics of the airscrew.

Two series of airscrews were designed. In the first series were four airscrews of diameter 9 ft. and of rotational speeds 1800 r.p.m., 1800 r.p.m., 1400 r.p.m., and 1000 r.p.m. respectively. The first airscrew of this series had two blades, each of the others four blades. In the second series were four airscrews each of diameter 10 ft. Two of these airscrews had two blades each and rotational speeds 1600 r.p.m. and 1200 r.p.m. respectively. The other two were four-bladers with rotational speeds of 1200 r.p.m. and 900 r.p.m. respectively.

We found that the performance of the four-bladed airscrew of diameter 9 ft. at the rotational speed of 1800 r.p.m. was very poor, because with this speed and diameter a four-bladed airscrew with blades of reasonable shape can only absorb 300 h.p. when the angle of incidence of the blade is small. On the other hand, with the four-bladed airscrew of diameter 9 ft. which was designed to rotate at 1000 r.p.m. the efficiency fell somewhat rapidly at the climb, because to absorb the given horse-power at a rotational speed of about 1000 r.p.m. the blades, even if wide in plan form, need to be working at a large angle of incidence. From a comparison of the performances of the airscrews of diameter 10 ft. we found that with a rotational speed of about 1200 r.p.m. the angle of incidence of the blade of a two-bladed airscrew was rather too high, and with a four-bladed airscrew rather too low, for good efficiency of working. The four-bladed airscrew of diameter 10 ft. and of rotational speed 900 r.p.m. was the best of the second series, although there was a tendency for the efficiency to fall somewhat rapidly at the climb because of the large angle of incidence at which the blades work in order to absorb the given horse-power. It is undesirable, then, to put more than 300 h.p. into a four-bladed airscrew of diameter 10 ft. at working speeds of 900 r.p.m. and 125 m.p.h.

From the performance data of the several airscrews designed to rotate at different speeds it was possible to consider whether gearing was desirable, and if so, the diameter of the airscrew and the ratio of the reduction gearing to give the best all-round performance of the aeroplane. In this connection it is well to point out that it is not sufficient to consider the relative merits of the airscrews when each airscrew is regarded independently. It is necessary to compare the performances of the combinations of the several airscrews and the same engine when account is taken of efficiency and weight of the reduction gearing. The conclusion we came to was that from the standpoint of airscrew design, reduction gearing was not necessary in the case of an engine developing 300 horse-power at 1800 r.p.m. at an altitude of 10,000 ft., the forward speed of the machine being 125 m.p.h. With such a rotational speed it should be noted that the diameter is limited to about 10 ft., otherwise the speed at the tip becomes excessively great.

Owing partly to this limitation of the diameter, when mounted directly on a high-speed engine, it is probable in the case of an engine developing a very high power at a high rotational speed, say 600 h.p. at 1600 r.p.m., that the employment of a reduction gearing would be needed if the aerodynamic design of the airscrew is to be performed efficiently.

(*b*) The Variable-Pitch Airscrew

If an airscrew with fixed blades be designed for maximum efficiency at the maximum horizontal flight speed of the aeroplane, and to hold the engine on full throttle at the maximum permissible revolutions, there is at the same altitude a loss of thrust horse-power at climbing due (*a*) to decreased revolutions of the engine, and (*b*) to decreased efficiency of the airscrew. An airscrew may, of course, be designed to give a good efficiency especially at climbing, which will probably mean a loss of efficiency when flying at maximum horizontal flight speed, or to give maximum revolutions at climbing, in which case the engine will need to be throttled at higher speeds. Hence an airscrew with fixed blades may only be designed for one definite set of working conditions. Generally speaking, the performance of a variable-pitch airscrew will be the envelope of the performances of airscrews which are designed for special purposes and may thus enable the over-all efficiency of the combination of airscrew and engine to be well maintained at all speeds of flight. Simply expressed, a variable-pitch airscrew, by keeping the rotational speed constant, enables the engine to develop its maximum horse-power over the range of flying speed, and at the same time enables the efficiency of the airscrew to be well maintained. With a variable-pitch airscrew the full engine power may be used during a steep dive without the engine rotating at an excessive speed. Also if the mechanical design permit, the airscrew may be used as a brake when the aeroplane alights on the ground. The greatest disadvantage of a variable-pitch airscrew would appear to be mechanical ; this is especially the case with engines of high power. Also the setting of the airscrew blades necessitates another control for the pilot.

There are two methods by which the pitch of an airscrew may be adjusted, (*a*) by rotating the blade as a whole and (*b*) by rotating a flap at the trailing edge of the airscrew.

A variable-pitch airscrew with which the whole of a blade may be rotated has been designed and constructed at the Royal Aircraft Establishment.* This airscrew was designed for the R.A.F. 1a engine of 100 h.p. mounted on the B.E. 2c aeroplane. The curves of Fig. 100, which show the relationship between the rate of climb and the forward speed of the B.E. 2c aeroplane, were drawn from the data of some experiments made at the Royal Aircraft Establishment with (*a*) the variable-pitch airscrew, (*b*) an airscrew designed for climbing by allowing the engine to reach its maximum revolutions at the climbing speed, and (*c*) a standard airscrew mounted on the R.A.F. 1 engine. When necessary, the engine was throttled to prevent the rotational speed exceeding the maximum

* The Variable-Pitch Propeller.—Experiments conducted at the Royal Aircraft Establishment. Advisory Committee for Aeronautics, 1918.

safe limit of 1800 r.p.m. With the variable-pitch airscrew the angle of the blade was adjusted to give the best climbing performance at each forward speed. With this airscrew it was found that if the rotational speed were left constant at 1800 r.p.m. the thrust horse-power did not change appreciably with a change forward speed of from 50 to 100 m.p.h. The curves of this figure show very clearly the aerodynamic advantages of the variable-pitch airscrew ; the performance at the climb is as good as that of an airscrew designed to be specially efficient at the climb, whilst the maximum speed of the aeroplane in horizontal flight is as great as that obtained with an airscrew designed to run most efficiently at this forward speed.

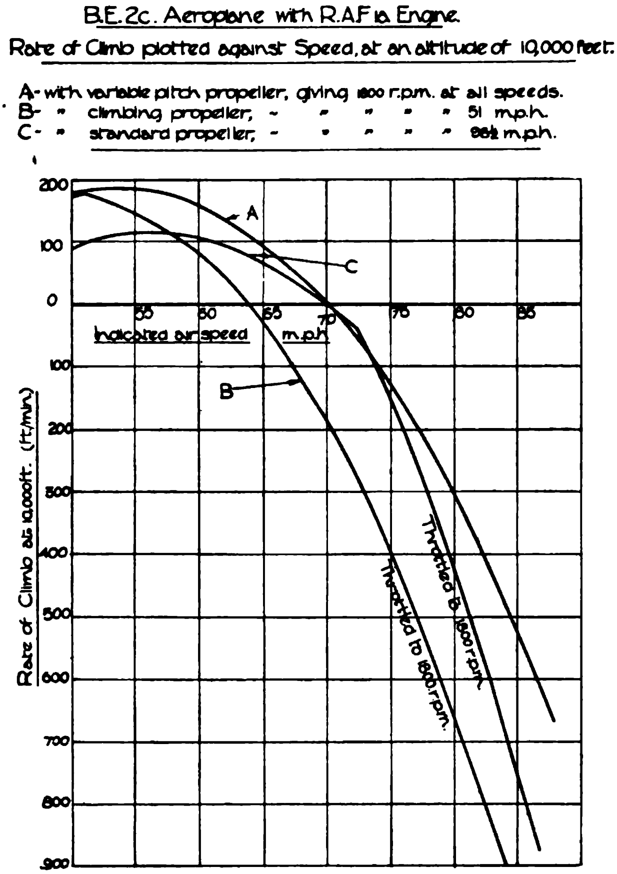


FIG. 100.

As previously stated, the mechanical working of a variable-pitch airscrew may occasionally give trouble. These experiments at the Royal Aircraft Establishment showed, however, that a variable-pitch airscrew which is both strong and easy to operate can be designed for a 100 h.p. engine.

Some few years ago a variable-pitch airscrew, the shape of each blade being alterable by means of an adjustable flap on the trailing edge, was designed by the Admiralty for use on a hydro-aeroplane. A sketch of the airscrew is given in Fig. 101, which shows how each blade section may assume a different shape by a partial rotation of the flap. The object of this design of airscrew was to allow the engine to rotate at a high speed and so to develop the thrust necessary to get the hydro-aeroplane off the water at a low translational speed. Accordingly the flaps were set at a small negative angle at a low translational speed and at small positive angles at the ordinary speeds of flight. The diameter of the full-scale airscrew was 12 ft. 6 in.

Experiments were made at the National Physical Laboratory* with two

* " Tests on a model of a two-bladed airscrew, the shapes of the blades being alterable by means of adjustable rear flaps," by A. Fage, A.R.C.SC., and A. Landells, B.SC. Advis. Comm. Aeron., 1915.

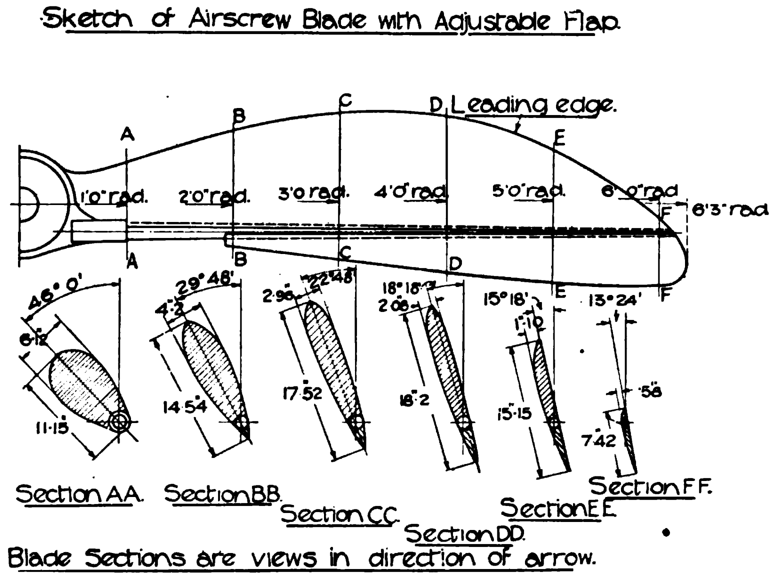


FIG. 101.

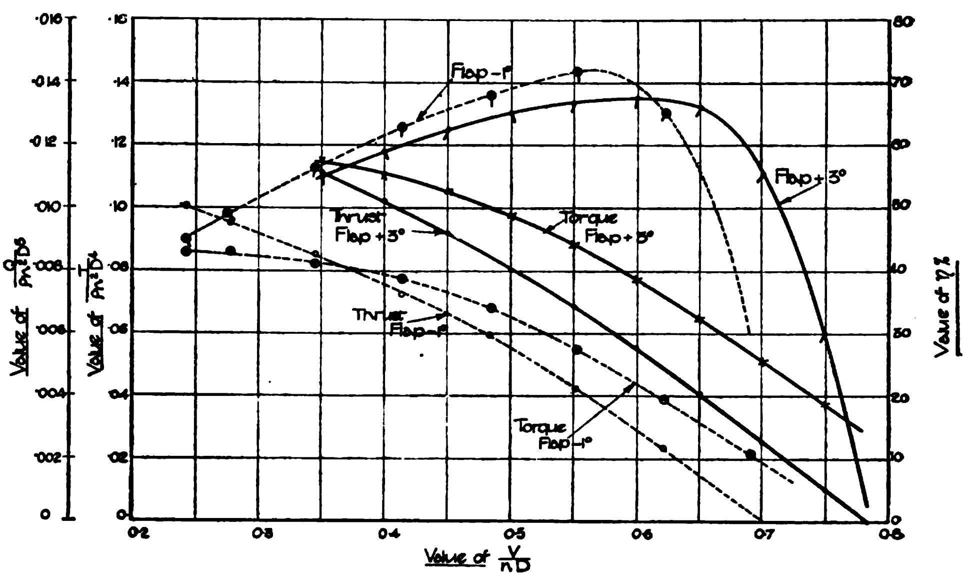


FIG. 102.

models of this airscrew, the flap with one model being at $+3°$ and with the other at $-1°$. The data of these experiments are shown graphically in Fig. 102. It is seen that a rotation of the flap from $-1°$ to $+3°$ increases the experimental mean pitch from 0·7D to 0·78D, and decreases the value of the maximum efficiency from 72·0 per cent to 67·5 per cent. It can easily be shown that at a low translational speed the airscrew develops the greater thrust when the flap is at $-1°$. This airscrew was designed for an engine which developed 215 h.p. at a rotational speed of 905 r.p.m., so that at this speed the torque absorbed by the airscrew would be 1235 lb.-ft. Assuming the torque of the engine to remain constant at all rotational speeds, the variation of the thrust with the forward speed of the airscrew has been calculated for each setting of the flap. The data of these calculations are shown in Table XIX, from which it will be seen that if the "hump" speed of this hydro-aeroplane be taken at 40 m.p.h.—at the "hump" speed the sum of the air and water resistances is a maximum—the thrust may be increased by 100 lb. by rotating the blade flaps from $+3°$ to $-1°$. It will also be noticed that the engine speed has been increased from 740 r.p.m. to 860 r.p.m.

TABLE XIX

Translational speed of hydro-aeroplane. Miles per hour.	Rotational speed of airscrew. Revolutions per minute.		Thrust of airscrew. lb.	
	Flap at $+3°$.	Flap at $-1°$.	Flap at $+3°$.	Flap at $-1°$.
0	720	810	1260	1360
40	740	860	940	1040
50	765	885	835	955
60	810	927	786	890
70	860	976	740	855
80	915	1035	685	790
90	980	—	640	755

(c) Forces on an Airscrew due to a Side-Slip or a Lateral Wind

Mathematical Theory.—It is of importance to know the magnitude of the forces acting on an airscrew when the motion of the aeroplane is other than rectilinear, since such forces may appreciably influence the stability of the aeroplane. The first investigations of the effect of a side wind on the performance of an airscrew were made by Clarke* of the Royal Aircraft Establishment and Bramwell† of the National Physical Laboratory. The methods, which were the same in principle but differed only in that one was analytical and the other graphical, were to calculate the lateral force in a manner similar to that used in the calculation of the performance of an airscrew, that is by the integration of

* "Effect of side wind on a propeller," by T. W. K. Clarke. Advis. Comm. Aeron., 1913.

† "Experiments to determine the lateral force on a propeller in a side wind," by F. H. Bramwell, B.SC., E. F. Relf, A.R.C.SC., and L. W. Bryant, A.R.C.SC. Advis. Comm. Aeron., 1914.

the forces acting on the elements of a blade. In this case, however, the integration needs to be carried out for several positions of the blade during a revolution because the magnitude and the direction of the resultant wind velocity on a blade element are functions of the angular position of the blade. A later theoretical investigation of the effect of a side-slip on the forces acting on an airscrew has been made by Harris.* The method adopted is of general applicability, the forces on the airscrew due to any manœuvre of the aeroplane being calculated directly from the ordinary performance curves. It is now proposed to present the subject as developed by Harris.

A diagrammatic sketch of an airscrew blade with the rectangular axes in position is shown in Fig. 103. The axis OX coincides with the airscrew axis, the positive direction being that of the forward motion of the aeroplane. The axis OZ is taken in the plane of rotation of the airscrew and may be considered to be in the vertical direction in so far as the present investigation is concerned. The remaining axis, OY, has the direction which makes the system of axes right-handed. The view on the left hand of Fig. 103 is obtained by cutting the blade by a plane at right angles to the blade axis OC. As will be seen from the figure, the angle between the blade axis OC and the axis OY is represented by A. The thrust and torque acting on the blade element at a radial distance r from the axis of rotation are represented by dT and dQ respectively.

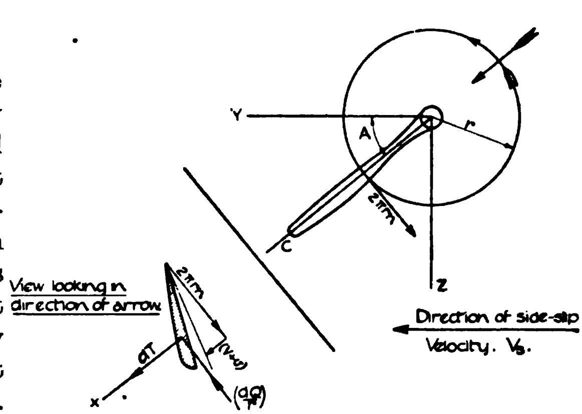


FIG. 103.

Hence the forces acting on the element of the blade are dT in the direction of the forward motion—that is axis OX—and (dQ/r) in a direction opposite to that of the rotation. For the present investigation it is convenient to write $dT = tdr = h$, and $dQ = qdr = k$. Also for any one airscrew $(t/\rho n^2)$ and $(q/\rho n^2)$ are functions of the advance per revolution (V/n), which will be denoted by p. Harris assumed that any small changes of the forces acting on the element of blade were due (1) to a change in the rotational speed, n, and (2) to a change in the advance per revolution, p.

$$\text{Hence } dh = \frac{\delta h}{\delta n}dn + \frac{\delta h}{\delta p}dp = \frac{2h}{n}dn + \frac{\delta h}{\delta p}dp$$

$$\text{and} \quad dk = \frac{\delta k}{\delta n}dn + \frac{\delta k}{\delta p}dp = \frac{2k}{n}dn + \frac{\delta k}{\delta p}dp,$$

since both h and k vary as n^2 when p is constant.

* "Forces on an airscrew due to a side-slip," by R. G. Harris. Advis. Comm. Aeron., 1917–18.

The corresponding elements of thrust, torque, lateral force, normal force, yawing moment, that is the couple about OZ, and pitching moment, that is the couple about OY, are dh, dk, (dk/r)sin A, —(dk/r)cos A, —dh.r.cos A, and dh.r.sin A respectively.

Denoting any one of the preceding quantities by Gdr, the average force or moment, as the case may be, is $\frac{1}{2\pi}\int_0^{2\pi}\int_0^{R} G.dr.dA$ for each airscrew blade.

Effect of a Side-slip on the Forces and Moments Acting on an Airscrew.—Assuming the aeroplane to be side-slipping with a velocity V_S. Then the increment in the rotational velocity is $(-V_S \sin A)$, which may be regarded as an increment of the rotational speed of magnitude $\left(-\frac{V_S \sin A}{2\pi r}\right)$. No account is here taken of the radial component $V_S \cos A$, which is of course similar in effect to a small angle of yaw on an ordinary aerofoil. On the other hand, the component $-V_S \sin A$ affects both the lift and the drag because of the small increments of both the rotational speed and the angle of incidence.

Effect of the Side-slip on the Thrust and Torque.—From the previous equations we have, since $dp = -\frac{p}{n}dn$

$$dh = \frac{dn}{n}\left[2h - p\frac{\delta h}{\delta p}\right] = \frac{-V_S \sin A}{2\pi rn}\left[2h - p\frac{\delta h}{\delta p}\right]$$

$$= \frac{-V_S \sin A}{2\pi rn}\left[2t - p\frac{\delta t}{\delta p}\right] dr$$

and $$dk = \frac{dn}{n}\left[2k - p\frac{\delta k}{\delta p}\right] = \frac{-V_S \sin A}{2\pi rn}\left[2k - p\frac{\delta k}{\delta p}\right]$$

$$= \frac{-V_S \sin A}{2\pi rn}\left[2q - p\frac{\delta q}{\delta p}\right] dr.$$

Hence $dh = C_1 \sin A$ and $dk = C_2 \sin A$, where C_1 and C_2 are constants throughout a revolution for any particular element. Also since $\int_0^{2\pi} \sin A.dA$ is zero it follows that the values of the thrust and the torque of the airscrew are unaffected by the side-slip.

Effect of the Side-slip on the Pitching Moment.—The increment of pitching moment is dh.r.sin A, that is

$$\frac{-V_S \sin^2 A}{2\pi n}\left[2t - p\frac{\delta t}{\delta p}\right] dr.$$

With a four-bladed airscrew $\Sigma \sin^2 A$ for the four similar elements is equal to 2 and also $\int_0^R t\,dr = (T/4)$ and $\int_0^R \left(\frac{\delta t}{\delta p}\right) dr = \frac{1}{4}\left(\frac{\delta T}{\delta p}\right)$, so that the pitching moment of the airscrew

$$= \frac{-V_S . n . \rho}{2\pi}\left[\frac{T}{\rho n^2} - \frac{p}{2} . \frac{\delta\left(\frac{T}{\rho n^2}\right)}{\delta p}\right].$$

Since $\left[\frac{T}{\rho n^2} - \frac{p}{2} . \frac{\delta(T/\rho n^2)}{\delta p}\right]$ is positive over the usual range of flying speeds it follows with this system of axes and with the airscrew rotating in the right-handed direction on an aeroplane which has a positive side-slip that the pitching moment is negative. This would of course be expected since the centre of thrust of the airscrew moves towards the tip of the blade which, because of the side-slip, is moving into the air with the greater velocity. It should be noted that the above expression is also the yawing moment due to a pitch of the airscrew.

With two blades, the value of $\Sigma \sin^2 A$ is periodic throughout the revolution and of average value 1/2. The average value of the pitching moment for the two-bladed airscrew is therefore

$$\frac{-V_S}{2\pi n}\left[T - \frac{p}{2}\frac{\delta T}{\delta p}\right], \text{ that is}$$

$$\frac{-V_S . n . \rho}{2\pi}\left[(T/\rho n^2) - \frac{p}{2} . \frac{\delta(T/\rho n^2)}{\delta p}\right],$$

which is the same as with a four-bladed airscrew. With the two-blader the maximum value of the pitching moment is twice the above value and occurs twice throughout a revolution, when the blades are vertical.

Effect of a Side-slip on the Lateral Force.—The increment of the lateral force on a blade element is $\frac{dk}{r} \sin A = \frac{-V_S \sin^2 A}{2\pi r^2 n}\left[2q - p\frac{\delta q}{\delta p}\right] dr.$

With a four-bladed airscrew the lateral force

$$= -\int_0^R \frac{V_S}{\pi r^2 n}\left[2q - p\frac{\delta q}{\delta p}\right] dr.$$

Before the above equation may be reduced to a more convenient form it is necessary to know how the torque absorbed by each element of the blade varies with the radial distance of the element from the axis of rotation. From several comparisons of the lateral force as measured, with the lateral force as calculated from the known torque grading curves, Harris found that the preceding expression for the lateral force may be written

$$\frac{-1 \cdot 8 \rho n V_S}{\pi R^2}\left[\frac{Q}{\rho n^2} - \frac{p}{2}\frac{\delta(Q/\rho n^2)}{\delta p}\right].$$

Over a greater part of the speed range $\left[\frac{Q}{\rho n^2} - \frac{p}{2}\frac{\delta(Q/\rho n^2)}{\delta p}\right]$ is positive, that is, the

lateral force has a negative value, so that the airscrew will be acting as a fin, no matter in which direction it is rotating. With a two-bladed airscrew the magnitude of the lateral force will be periodic. In this case the maximum value is twice the average and occurs twice during a revolution. Some experiments made at the National Physical Laboratory show that at a constant value of the thrust, the lateral force is proportional to (V_S/V), which is an approximate value of the angle of yaw.

Effect of a Side-slip on the Normal Force and the Yawing Moment.—The increment of normal force on a blade element is $-\frac{dk}{r}\cos A$, that is

$$\frac{+V_S \cos A \sin A}{2\pi r^2 n}\left[2q - p\frac{\delta q}{\delta p}\right] dr.$$

The increment of yawing moment, that is the moment about OZ, on a blade element is $-$ dh.r.cos A, that is

$$\frac{V_S \sin A \cos A}{2\pi n}\left[2t - p\frac{\delta t}{\delta p}\right] dr.$$

With a four-bladed airscrew it follows, then, that there is no normal force or yawing moment at any time. With a two-bladed airscrew both the normal force and the yawing moment will be of a fluctuating magnitude, but the average values will be zero. The two maximum and the two minimum values of the normal force and the yawing moment during each revolution of a two-bladed airscrew occur when the blades are in the diagonal positions.

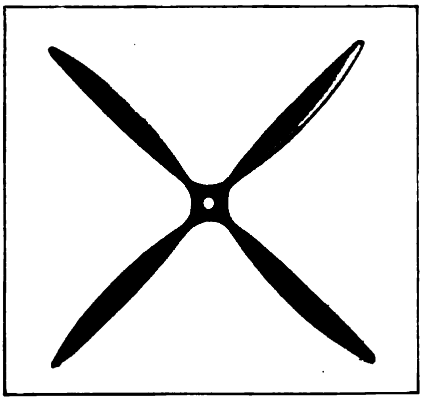


FIG. 104.

A Practical Illustration of the Theory.—To illustrate the preceding theory it is now proposed to calculate the pitching moment and the lateral force at various angles of yaw for a particular airscrew. The photograph of the airscrew for which the calculations are to be made is shown in Fig. 104. The performance curves are given in Fig. 105. This four-bladed airscrew of diameter 8 ft. 10 in., which was mounted on the B.E. 2 aeroplane, was designed to absorb about 72·5 h.p. at a translational speed of about 80 m.p.h. and a rotational speed of 800 r.p.m. The curves of Fig. 106 were drawn from the performance data of Fig. 105, and show how the values of

$$\left[\frac{T}{\rho n^2} - \frac{p}{2}\left(\frac{\delta(T/\rho n^2)}{\delta p}\right)\right] \text{ and of } \left[\frac{Q}{\rho n^2} - \frac{p}{2}\frac{\delta(Q/\rho n^2)}{\delta p}\right]$$

vary with the value of p, that is the forward advance per revolution.

The data of Table XX show for two working conditions of the airscrew how the values of the pitching moment and the lateral force vary with the angle o

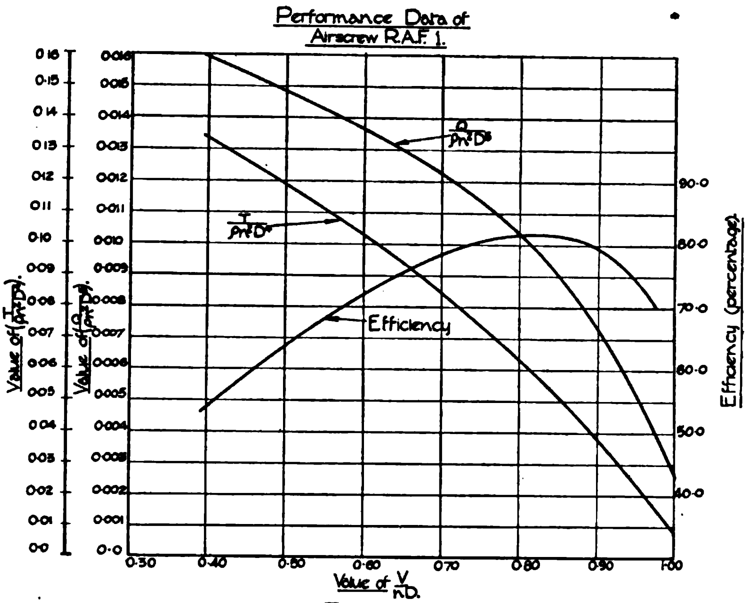


FIG. 105.

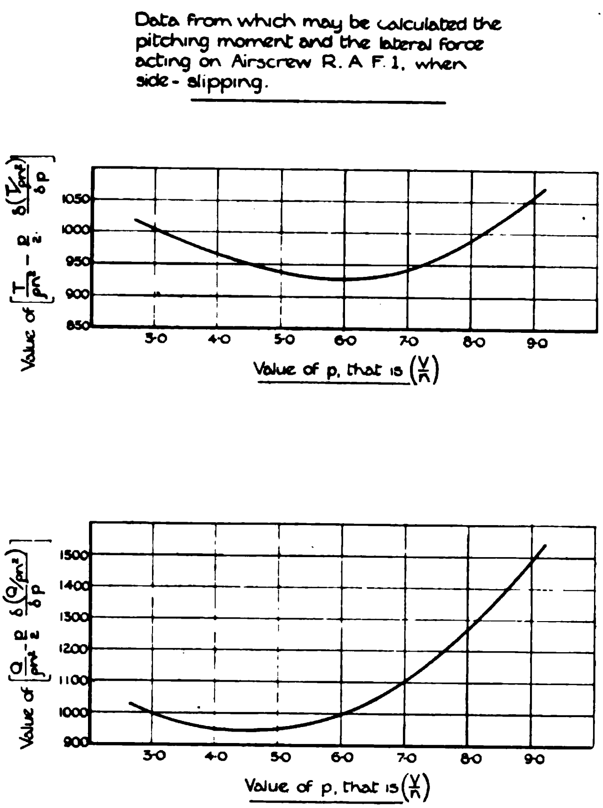


FIG. 106.

yaw, that is (V_S^*/V) approximately. It is seen that both the lateral force and the pitching moment decrease as the forward speed of the aeroplane decreases. The lateral forces for each degree of yaw at forward speeds of 80 m.p.h. and 50 m.p.h. are 1 per cent and 0·3 per cent of the thrust respectively. With this airscrew the centre of thrust is shifted about 6 inches when the airscrew is yawed through 15°.

TABLE XX

Working conditions of the airscrew.	Angle of yaw (degrees).	Value of the velocity of side-slip, V_S. ft./sec.	Pitching Moment lb.-ft.	Shift of centre of thrust expressed as a percentage of the diameter.	Lateral force	
					lb.	Percentage of thrust.
V = 80 m.p.h. = 117·2 ft./sec. n = 1050 r.p.m. = 17·5 r.p.s. p = V/n = 6·70. T = 315 lb.	5 10 15	10·3 20·7 31·4	−63·5 −132·0 −200·0	−2·3 −4·8 −7·2	−13·5 −27·0 −41·0	−4·5 −8·5 −13·0
V = 50 m.p.h. = 73·3 ft./sec. n = 925 r.p.m. = 15·4 r.p.s. p = V/n = 4·75. T = 390 lb.	5 10 15	6·4 12·9 19·6	−35·0 −71·0 −107·5	−1·0 −2·1 −5·1	−6·5 −13·2 −20·0	−1·5 −3·5 −5·0

(*d*) Effect of a Side Wind on the Performance of an Airscrew which has no Forward Motion

Occasionally it is necessary to know how the thrust and the torque of an airscrew, which has no forward motion, are modified by a lateral wind, that is a wind at right angles to the axis of the airscrew. Such a case would be that of an airscrew swivelled through 90° from the direction of forward motion, so as to give a vertical lift on an airship. The performance of a model airscrew when rotating at a stationary point in a wind at right angles to the airscrew axis has been measured at the National Physical Laboratory.* A photograph of the airscrew is given in Fig. 104. The data of these experiments are shown graphically in Fig. 107, from which it will be seen that both the absolute coefficients of thrust and torque increase with (V_S/nD) when V_S represents the magnitude of the side wind.

From a practical standpoint it is perhaps of more interest to know how the thrust of the airscrew varies with the magnitude of the side wind when the torque absorbed is constant. Accordingly, then, the airscrew of diameter 8 ft. 10 in. is assumed to be mounted on an engine with which the throttle is so adjusted that

* "Test of an airscrew with the axis of rotation at right angles to the wind direction," by E. F. Relf. Advis. Comm. Aeron., 1915–16.

72·5 h.p. is absorbed at the ground level by the airscrew when rotating, without a side wind, at a stationary point. Under such conditions the airscrew rotates at 850 r.p.m. and absorbs a torque of 450 lb.-ft. Keeping the same setting of the engine throttle, and assuming that a constant torque is then absorbed by the airscrew, it is a simple matter to calculate from the data of Fig. 107 how the thrust varies with the speed of the lateral wind. The data of such calculations are shown in Fig. 108, from which it will be seen that the thrust of the airscrew—which has no motion in the axial direction—is practically unaffected by a lateral wind. The maximum fluctuation over a wind range of 80 m.p.h. is only about 5 per cent of the static thrust, i.e. when $V_S=0$. It will also be noticed that the rotational speed falls continuously with an increase of the speed of the lateral wind. With this airscrew the value of (T/H.P.) increases moderately slowly with an increase of V_S.

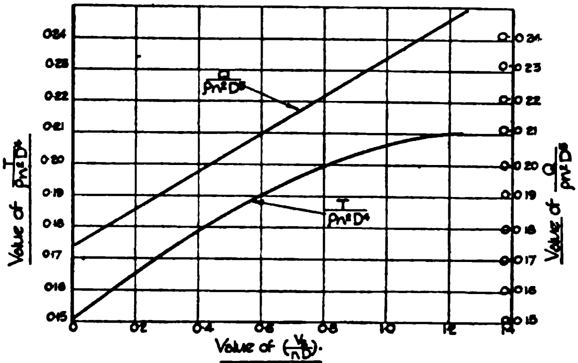


FIG. 107.

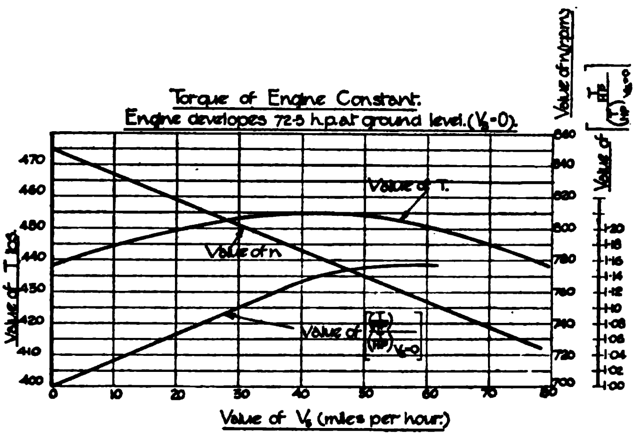


FIG. 108.

According to Riabouchinsky* the value of (T/H.P.) would appear to increase with an increase of V_S at a more rapid rate than that found from the above experiments. It should, however, be pointed out that Riabouchinsky made his experiments with an airscrew designed to rotate at a stationary point—angle of blade about 6°—and not with an ordinary airscrew.

* " Recherches sur l'hélice aérienne se mouvant dans un courant aérien dirigé perpendiculairement à l'axe de l'hélice," D. Riabouchinsky. " Aérodynamique de Koutchino," Part I, 1906.

(*e*) THE PERFORMANCE OF AN AIRSCREW ROTATING AS A WINDMILL

Some experiments* were made at the National Physical Laboratory to obtain data from which to calculate the resistance of an airscrew, when rotating as a windmill, with the engine shut off and the aeroplane in gliding flight. A photograph of the four-bladed airscrew with which the experiments were made is shown in Fig. 104. Each airscrew blade was long and narrow, the ratio of the length to the maximum chord-length being about 9·0. The airscrew, of diameter 8 ft. 10 in.,

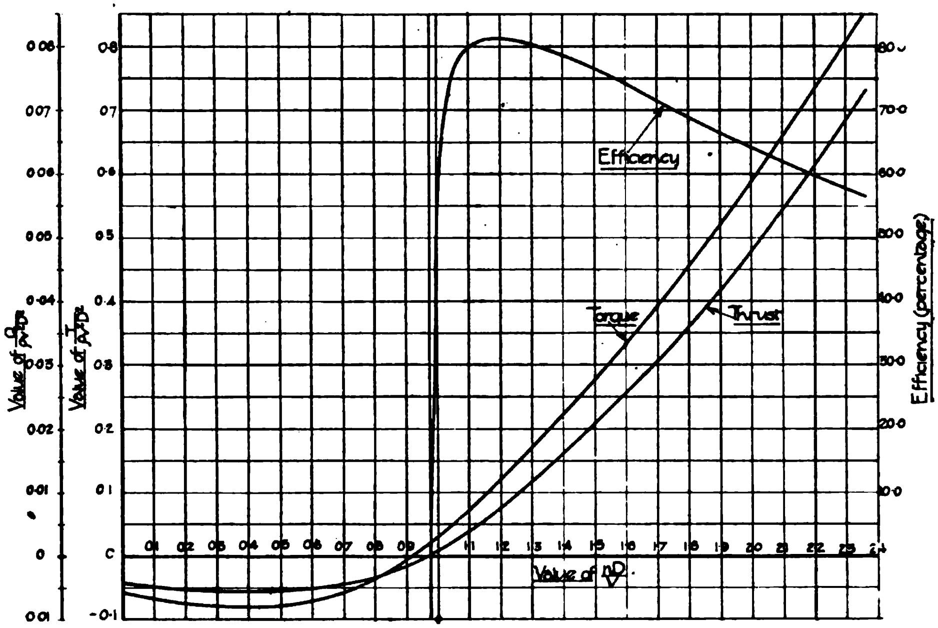


FIG. 109.

was designed for an engine of 70 h.p. mounted on the B.E. 2 aeroplane (1911). For convenience of plotting, the performance curves (see Fig. 109) are given in terms of absolute coefficients ($T/\rho V^2D^2$), ($Q/\rho V^2D^3$), and (nD/V), because measurements were made with the airscrew not rotating, in which case the ordinary absolute coefficients ($T/\rho n^2D^4$) and ($Q/\rho n^2D^5$) would each be infinite. It will be seen that this airscrew has a maximum efficiency of the high value of 81·2 per cent. The probable efficiency at the average working value of (V/nD) is 76·5 per cent. It should be noticed that the maximum negative values of the thrust and torque coefficients do not occur when (nD/V)$=0$. This is in agreement with the data of some experiments made at the Aerodynamic Laboratory at Koutchino. The performance curves of Fig. 109 are of special interest because from them the resistance of the airscrew in gliding flight may be calculated. It

* "Tests on a four-bladed airscrew for thrust and efficiency, the experiments being extended to negative values of the thrust to determine the resistance during gliding flight," by F. H. Bramwell, B.SC., and A. Fage, A.R.C.SC. Advis. Comm. Aeron., 1913.

was found from some full-scale experiments at the Royal Aircraft Establishment with the B.E. 2 machine that a torque of about 80 lb.-ft. was expended in overcoming the mechanical and frictional losses of the engine. Using this value of the resisting torque and the data of Fig. 109 it is a simple matter to calculate the resistance of the airscrew during gliding. The data of such calculations are shown graphically in Fig. 110. As the gliding speed decreases the rotational speed also decreases, until the forward speed becomes 53·3 m.p.h. and the rotational speed 185 r.p.m. At these speeds of rotation and translation the absolute torque coefficient has the maximum negative value of —0·008. It is easily seen that it is at the forward speed of 53·3 m.p.h. that the airscrew ceases to rotate.

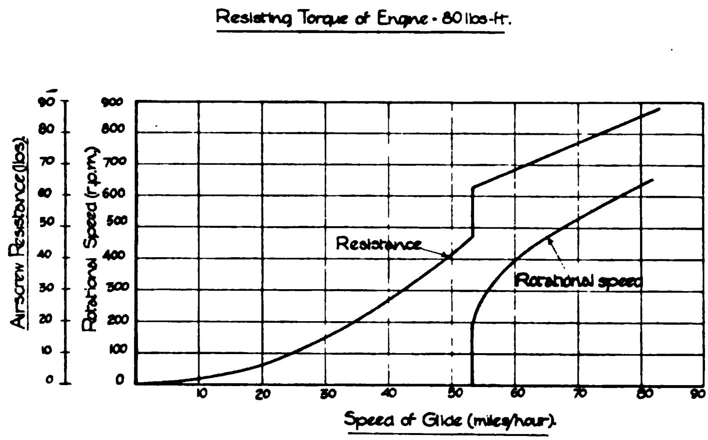


FIG. 110.

Thus, if at this speed the rotational speed be increased—by some external agent —the " windmill " torque will be smaller than the resisting torque of the engine, so that the rotational speed decreases. On the other hand, at this forward speed and any rotational speed less than 185 r.p.m. the " windmill " torque is less than the resisting torque, so that the airscrew if left to itself ceases to rotate. To start the airscrew again the aeroplane would of course need to glide at

$$\sqrt{\left(\frac{80 \times (60)^2}{\cdot 00237 \times (8 \cdot 83)^3 \times \cdot 0056 \times (88)^2}\right)},$$

that is 63·7 m.p.h.

It is also seen from Fig. 109 that the resistance of the airscrew during gliding flight is large. Assuming the thrust of the airscrew at the normal flying speed of 80 m.p.h. to be 280 lb., the resistance of the airscrew with the aeroplane gliding at 53·3 m.p.h. is about 17 per cent of the thrust at the normal flying speed.

(*f*) THE EMISSION OF SOUND FROM AN AIRSCREW

The advantages of silencing aircraft, whether used for war or commercial purposes, are obvious. The noise of an aeroplane in flight is emitted from both the airscrew and the engine. Probably the greater part of the steady roar is

emitted from the airscrew, whilst the throbbing noise is due to the periodic discharge of the exhaust gases from the engine. As would be expected, the throbbing is generally more pronounced with a rotary engine, in which case the exhaust gases are discharged from rotating cylinders. Lanchester has suggested that the noise emitted from an airscrew is probably due to the movement of centres of pressure of approximately constant magnitude in a helical path. With the motion of each blade a high-pressure and a low-pressure pair are generated—source and sink—the high-pressure centre being situated some distance from the face of the blade and the low-pressure centre at the back of the blade; the sound wave being generated by the movement of this constant pressure difference in a helical path. The intensity of the sound emitted, which would appear to depend on the change of motion of the pressure centres and also on the velocity of sound, increases with an increase of the tip speed of the blade. Lanchester has further suggested that, since the intensity of the sound emitted from slow-running airscrews is comparatively low, the noise emitted from an aeroplane could be greatly lessened by the employment of airscrews of high pitch and large diameter, but of low tip speed. To develop the necessary thrust airscrews of opposite direction of rotation, mounted in tandem, could be used. In view of the low rotational speed of each airscrew as compared with that of the engine a reduction gearing would be needed, which should preferably be of the silent-running epicyclic type.

It has been suggested by Griffith that some of the noise emitted from an airscrew may be partly due to an "out-of-balance" twist of the blades. Assuming the density of the air in the neighbourhood of the blade to be a function of the blade angle, and also that the angles of corresponding parts of the blades differ, it would seem that a constant stream of condensations and rarefactions would be coming away from the airscrew disc, that is sound would be emitted.

The variation of the intensity of the sound emitted from an airscrew with the speed of the blade tip has been investigated mathematically by Lynam* and Webb. The problem was limited to the case of airscrews of the same diameter developing the same thrust at the same forward speed, but at different rotational speeds. The theory takes account of the compressibility of the air, but the motion is assumed to be irrotational. It is pointed out by these writers that the noise emitted from an airscrew on an aeroplane in flight may be due to one or all of the following causes :—

(1) Movement of pressure centres of constant or nearly constant magnitude in circular orbits.

(2) Vibration of the airscrew blades. It is common knowledge that airscrews with flexible blades are usually very noisy.

(3) The periodic passing of the airscrew blades in close proximity to other parts of the aeroplane.

Considering only the first cause, Lynam and Webb found that the intensity

* "The emission of sound by airscrews," by E. J. Lynam and H. A. Webb. Advis. Comm. Aeron., 1918.

of the sound emitted increased continuously with an increase of the tip speed. Further, there would appear to be no discontinuity of the sound emitted when the tip speed approaches the velocity of sound in air. It is shown that an airscrew with two blades is much noisier than one with four blades at ordinary tip speeds. In both cases the maximum noise would appear to occur in the plane of rotation. Although the investigation is very ingenious, this method of calculation should be accepted with some reserve, because many assumptions need to be made in the theoretical treatment of such a difficult problem.

Some experiments with an airscrew rotating at a high tip speed have been made by Lynam* at the Royal Aircraft Establishment. He found that there was no perceptible discontinuity of the intensity of the sound emitted as the tip speed of the airscrew blade approached and exceeded the velocity of sound in air. The maximum intensity of sound appeared to occur in or slightly in front of the plane of rotation. Here the noise was of the nature of a scream, as compared with a roar of lower frequency elsewhere. The noise was a minimum both in front of and behind the airscrew disc, so that it would appear that no waves are propagated along lines at right angles to the plane of rotation. The sound heard in front of and behind the airscrew disc is probably due to both diffraction and reflection from surrounding objects of the sound waves propagated in other directions. According to Lynam, the noise in the plane of rotation has a peculiar and indescribably unpleasant physiological effect when the tip speed of the blade is equal to that of sound.

* "Preliminary report on experiments with a high tip-speed airscrew at zero advance," by E. J. Lynam, A.R.C.SC. Advis. Comm. Aeron., 1919.

CHAPTER XII

HELICOPTERS AND WINDMILLS

(a) HELICOPTERS

Theory.—At present there is no reliable theory of a helicopter. This is probably due, firstly, to the want of knowledge of the working régime, and, secondly, to the extreme difficulty of expressing the rather meagre experimental knowledge in a concise but adequate mathematical manner. Undoubtedly, the study of the working régime of a helicopter affords ample scope for both experimental and theoretical investigation. We have already seen that a deficiency of the ordinary aerofoil theory of an airscrew is the uncertainty of the magnitude of the inflow velocity of air. It will readily be realised, then, that the difficulty of estimating the inflow velocity is much greater with a helicopter because, since the forward speed is zero, all the air passing through the airscrew disc is drawn in by the thrust. When theoretical difficulties of such a nature are considered it would seem that any theory to be of value would need to be of an empirical character, and should be based on a conception suggested by experiment. It is not to be expected that a theory based fundamentally on theoretical considerations could take complete account of the complicated air-flow around a helicopter. Assuming the Froude conception to be applicable to a helicopter we have $T=\frac{\pi D^2}{4}\rho 2(aV_0)^2$, and also the horse-power absorbed $=\frac{TaV_0}{550}$, where aV_0 is the inflow velocity of air measured in feet per sec.

$$\text{Hence } (T/H.P.)=\frac{550}{aV_0}=550\sqrt{\left(\frac{\pi D^2.\rho.2}{4.T}\right)}.$$

Since, however, the assumptions of Froude are not directly applicable to practice, the horse-power needed to develop a given thrust is greater than would be calculated from this equation. Lanchester has suggested that the deficiencies of the Froude Theory may be taken into consideration by the introduction of a factor such as K_t, where $(T/H.P.)=550\ K_t\sqrt{\left(\frac{\pi D^2.\rho.2}{4.T}\right)}$. This factor K_t, which has no dimension, may be regarded as a criterion of efficiency, in that it is a numerical measure of the extent to which the actual performance approaches that of a helicopter working under ideal conditions. With similar helicopters K_t has a constant value, so that the horse-power needed to develop

a given thrust will vary inversely as the diameter. Also if with similar helicopters the thrust be constant, n^2D^4 will also be constant, so that the horsepower needed to develop a given thrust will also be proportional to $\sqrt{n}$.

Experiments with Model Helicopters.—It is now proposed to consider in some detail the data of experiments made by Fage* and Collins to determine the dependence of the performance of a helicopter on the number, the shape, and the angle of the blades. The over-all diameter of each model helicopter, with each blade clamped at any desired angle, in an arm projecting from the boss, was

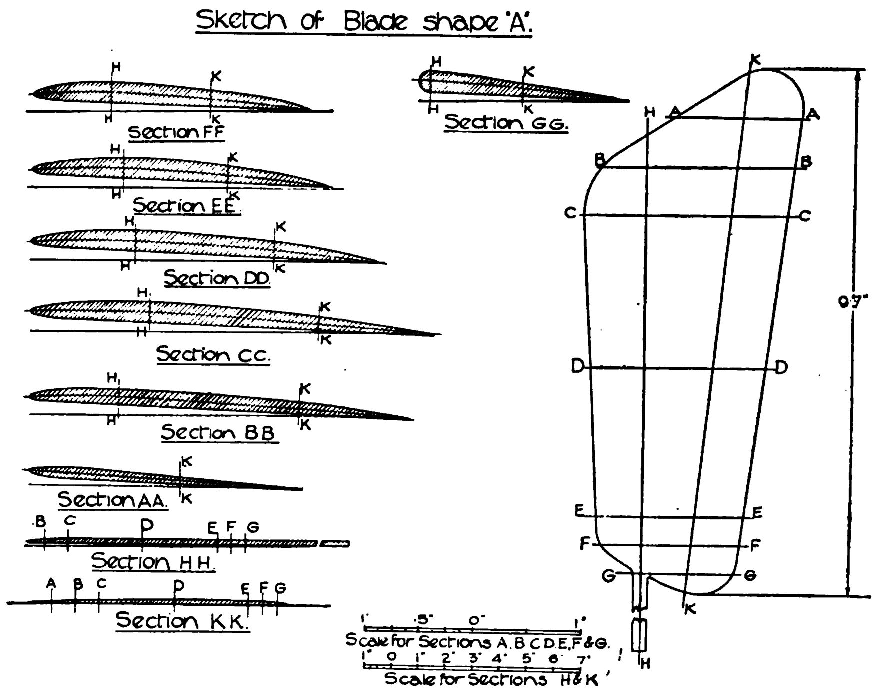


FIG. 111.

2·5 ft. Two types of blade, " A " and " B," were used in the experiments. The blade of shape " A," of which a sketch is given in Fig. 111, had a flat undersurface with the chords of the blade sections parallel to each other. With shape " B " (see Fig. 112) the under-surface was concave over a greater part of the blade, with the chords of the blade sections approximately parallel to each other. The performances were measured of helicopters with four blades, three blades, and two blades, respectively, each blade having the shape " A," when the blade angle was progressively varied from 1·5° to 16°. Also the ordinary experiments were made with a four-bladed helicopter with blades of shape " B," rotating at a stationary point, with a variation of the blade angle from

* " Some experiments on helicopters," by A. Fage, A.R.C.SC., and H. E. Collins. Advis. Comm. Aeron., 1917.

2·5° to 15·5°. To investigate the effect of either an up or down vertical gust, experiments were made with a four-bladed helicopter with a blade angle of 9·9° in translational motion, the value of (V/nD) ranging from −0·2 to +0·3.

The data of the experiments are given in Figs. 113–117. It should perhaps be stated that these data do not include the forces acting on the framework supporting the blades. With the angle range of the experiments the values of both ($T/\rho n^2D^4$) and ($Q/\rho n^2D^5$) increase with an increase of the blade angle. Also, as the blade angle increases, the value of K_f, which is very small at a low

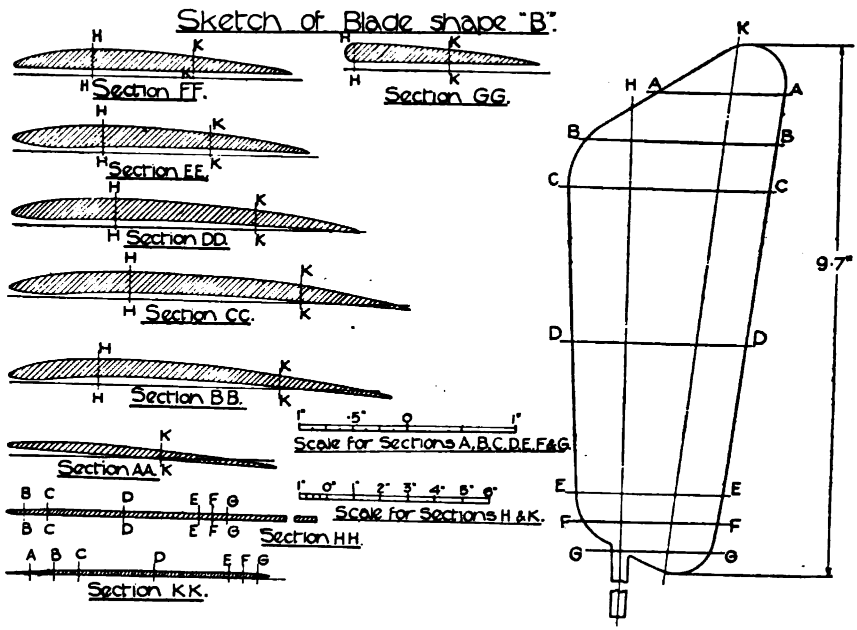


FIG. 112.

value of the blade angle, increases rapidly at first, reaches a maximum, and then slowly decreases. The angle at which the maximum value of K_f occurs is a function of the number of blades. Thus the values of this angle are 10°, 11°, and 12° with a two-bladed, a three-bladed, and a four-bladed helicopter respectively. With each helicopter the maximum value of K_f is about 0·55. At the blade angle of the maximum value of K_f it is probable that the blades—no matter what the number—are working at or near the angle of incidence which gives the maximum aerodynamic efficiency of the aerofoil section, so that an increase of the blade angle, at which the maximum value of K_f occurs, with the number of blades is to be expected, since the inflow velocity at the same rotational speed and blade angle increases with the number of blades. The data of Table XXI, which have been taken from Figs. 113–115, show very clearly that at constant values of both the blade angle and rotational speed the thrust of a blade diminishes with an increase of the number of blades. The reason is, of course, that with any

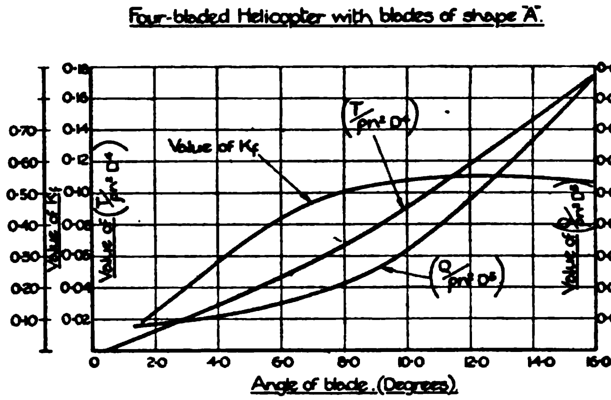


FIG. 113.

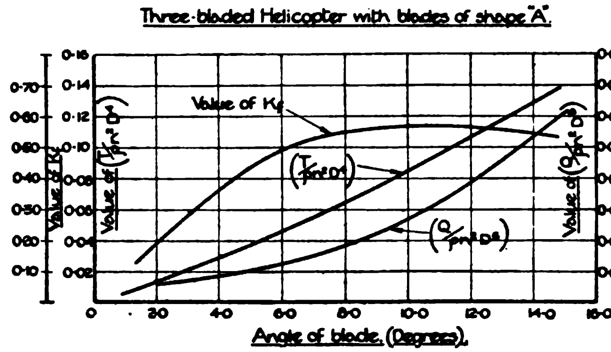


FIG. 114.

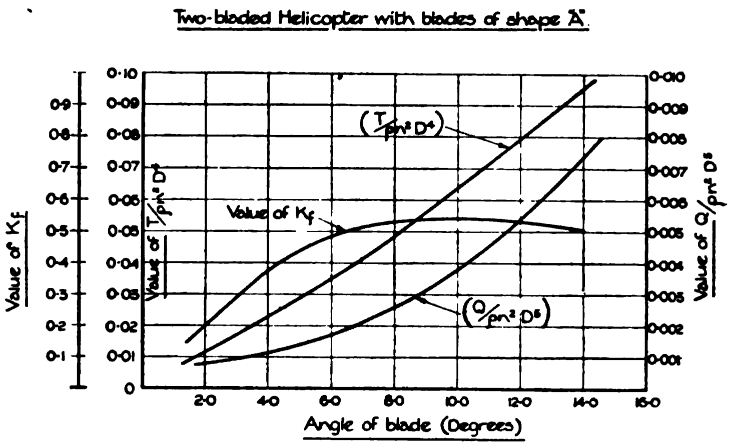


FIG. 115.

blade angle both the thrust of the helicopter and the inflow velocity of the air increase, and consequently the angle of incidence and the thrust per blade decrease, with the number of blades.

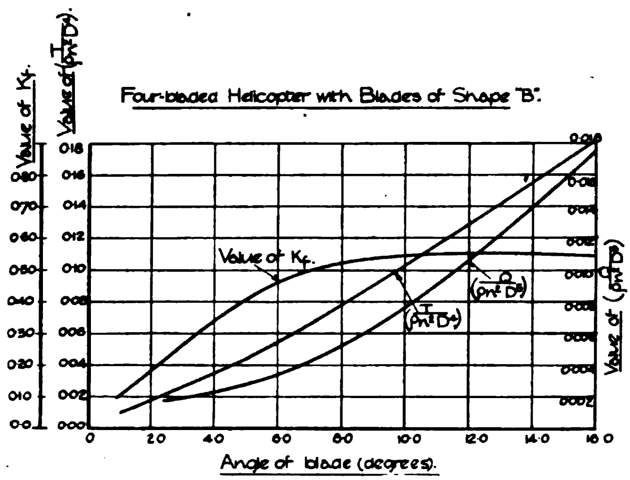


FIG. 116.

TABLE XXI

Angle of blade degrees.	Value of $(T/\rho n^2D^4)$ for each blade of shape "A" of the			Value of $(Q/\rho n^2D^5)$ for each blade of shape "A" of the		
	4-bladed helicopter.	3-bladed helicopter.	2-bladed helicopter.	4-bladed helicopter.	3-bladed helicopter.	2-bladed helicopter.
6	0·0115	0·0154	0·0176	0·00075	0·00087	0·00086
8	0·0165	0·0218	0·0244	0·00108	0·00124	0·00131
10	0·0228	0·0286	0·0319	0·00160	0·00178	0·00188
12	0·0295	0·0357	0·0400	0·00238	0·00356	0·00270
14	0·0365	0·0429	0·0478	0·00333	0·00353	0·00373

Behaviour of a Helicopter in a Vertical Wind.—The curves of Fig. 117 show for the four-bladed helicopter, with blades of shape "B" and of angle 9·9°, how the values of $(T/\rho n^2D^4)$, $(Q/\rho n^2D^5)$, and K_t vary with the value of (V/nD). It will be noticed that the maximum values of these quantities occur when the helicopter is descending with such a velocity that the value of (V/nD) is about $-0{\cdot}1$. It is of some interest to consider the behaviour of this helicopter in a gusty vertical wind. This subject can, of course, be only fully considered when the actual working conditions are known. For the purpose of illustration we shall assume that the helicopter has an over-all diameter of 40 ft., and that a weight of 1600 lb. is supported in still air at ground level. With such conditions of working the helicopter will be absorbing a torque of 4725 lb.-ft. at a rotational speed of 96 r.p.m. Parenthetically, these are the approximate working con-

ditions at which the helicopter was designed to work. Assuming the torque absorbed by the helicopter to be maintained constant at 4725 lb.-ft. the variation of the thrust with a forward speed—either positive or negative—may be easily calculated. The data of such a calculation are shown graphically in Fig. 118. It will be seen from this curve that if the helicopter rotating at a stationary point in still air at the assumed working conditions, be suddenly acted on by a downward vertical gust, the thrust developed will be less than the total weight, so that the helicopter falls until it has the same speed as the downward wind. If, on the other hand, an upward air current be encountered the helicopter rises as long as the wind velocity is less than 7 m.p.h. When, however, the downward motion of the helicopter relative to the surrounding air is greater than 7 m.p.h., the thrust is smaller than the weight to be supported, so that the helicopter falls with increasing velocity. Although the conditions of practice would probably be less rigid than those assumed—both the blade angle and the torque could be adjustable—the preceding calculation shows that the stability of a helicopter needs consideration.

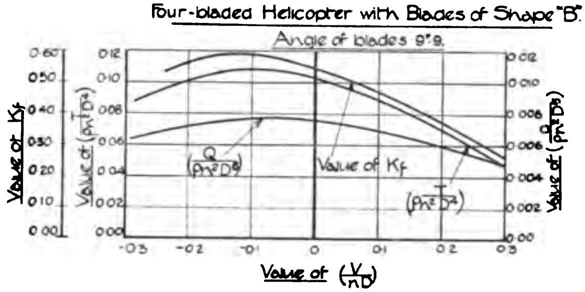


FIG. 117.

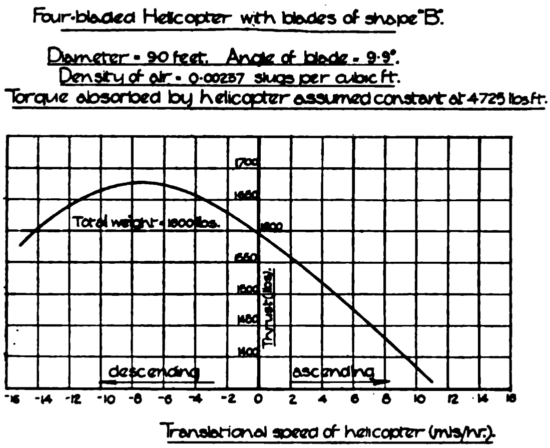


FIG. 118.

Blade Shape.—A systematic experimental investigation of the influence of blade form on the thrust and torque of an airscrew rotating at a stationary point has been made by Schmid.* The data of these experiments are presented in terms of the absolute coefficients (T/ω^2R_4), (Q/ω^2R^5), and (TR/Q), where ω is the rotational speed, of the helicopter. Schmid shows that a machine of the helicopter type cannot have the same economy of flight as that of an aeroplane. Also to develop a high thrust at a given horse-power, the position of the maximum ordinate of each blade section should be so adjusted that a maximum negative pressure is developed on the suction face and a minimum positive pressure on the pressure face ; suction should, however, be avoided on the latter face. The conclusion is made that it is the camber of the suction surface which is of impor-

* "Die Luftschraube am Stand," C. Schmid. Dissertation, Zeitschrift, für Flugtechnik und Motorluftschiffahrt. June, 1915.

tance. Schmid found that a wide variation of the camber of the suction face towards the trailing edge only slightly influenced both the thrust and the torque. He suggests, however, that when designing, care should be exercised to avoid shallow curvature of the profile of the suction face in the neighbourhood of the leading edge. The position of the maximum ordinate suggested is from 1/3 to 1/5 of the chord from the leading edge. A good inclination to the chord of the tangent to the suction face at the trailing edge is from 15° to 20°. With the pressure face 1/50 to 1/40 of the chord length is considered a suitable camber. With the suction face a camber of from 1/10 to 1/15 gives good experimental results. The actual type of curved surface of the pressure face is of small importance. Finally a well-rounded leading edge and a not unduly sharp trailing edge are recommended. Schmid found that with blades of geometrically similar profile the thrust of the helicopter increased somewhat more rapidly than the square root of the number of blades, whilst the torque increased approximately as the number of blades. Investigating the air-flow around a helicopter he found that air was drawn into the disc in a converging stream. Behind the helicopter the contraction of sectional area continued, spiral eddying motions forming the boundary of the outflowing stream. At the blade tips strong vortices were formed.

(*b*) Windmills

The working régime of a windmill is the reverse of that of an ordinary airscrew. In the latter case the energy absorbed from the engine reappears, with some loss, as the useful work done by the thrust of the airscrew. In the former case the energy of forward motion—such energy may be either the work done in pushing a windmill through still air, or if the windmill be rotating at a stationary point the kinetic energy of translation taken from the wind, or, of course, a combination of both—reappears, in part, as the useful work done by the torque. It is to be expected, then, that the theory of a windmill would be very similar to that of an airscrew, since in the one case the resistance created by the forward motion is accompanied by a useful torque, whilst in the other the applied torque is converted by the working régime into a useful thrust. The present theories of the working of a windmill are developed on lines suggested by both the aerofoil and momentum theories of an airscrew.

At present windmills of small horse-power can be advantageously employed on an aeroplane for driving the generator of a wireless set or a small pump. With aeroplanes used for experimental purposes, windmills have been used to supply the motive power for a recording apparatus and to work a cinema camera.

A Momentum Theory of a Windmill.—We have already seen from the ordinary momentum theory that an increase of velocity of the outflowing stream of an airscrew is a concomitant effect of the thrust; also that one-half of this increase of velocity occurs at the front of the airscrew. The working régime of a windmill is such, however, that a resistance is offered to the surrounding air. We should expect, then, a "slowing up" of the velocity of the air both in front and behind the disc of a windmill. The present theory of a windmill is developed in a manner

which is immediately suggested by the momentum theory of Froude. Firstly, we shall consider the case of a windmill rotating at a stationary point in a current of air, which at a sufficient distance from the windmill has a uniform velocity, V, and a uniform pressure, p. It is assumed that the air is both inviscid and incompressible. Also, since the windmill is regarded as a surface of instantaneous change of pressure, that there are no rotational motions of the air to be considered.

A diagrammatic sketch of the type of flow assumed is shown in Fig. 119.

Let $(1-a_1)V \equiv$ the velocity of the air—which is assumed uniform—immediately in front of the windmill.

and $(1-a_1b_1)V \equiv$ the velocity of the air—which is assumed uniform—at the section " 1 " where the pressure is uniform and of magnitude p.

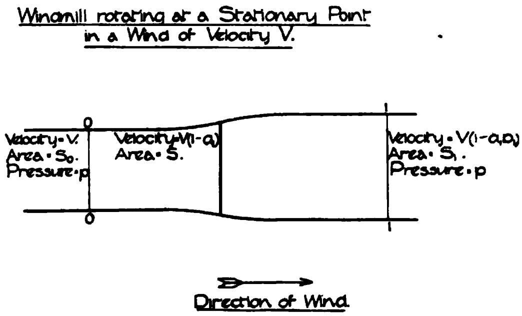


FIG. 119.

Adopting the conception of Froude, that the shapes of the inflowing and outflowing streams are such that the mean pressure over the boundaries is p, we have when equating for the equilibrium of the column of air passing through the windmill disc,

$$R = \rho S(1-a_1)V^2 a_1 b_1,$$

where R = resistance of the windmill.

Further, assuming the air-flow of either the inflowing or the outflowing stream to be such that the total head remains constant, we have :—
The pressure immediately in the front of the windmill disc

$$= p + \frac{\rho V^2}{2}[1-(1-a_1)^2],$$

and the pressure immediately at the back of the windmill disc

$$= p + \frac{\rho V^2}{2}[(1-a_1b_1)^2-(1-a_1)^2].$$

Hence the difference of pressure between the front and the back surfaces of the windmill disc

$$= \frac{\rho V^2}{2}[1-(1-a_1b_1)^2].$$

Hence $\quad R = \frac{\rho V^2 S}{2}.a_1b_1[2-a_1b_1].$

But $\quad R = \rho V^2 S[1-a_1]a_1b_1.$

Hence $b_1 = 2$, that is one-half the decrease of the velocity of the outflowing stream occurs in front of the windmill.

The kinetic energy of translation taken out of the air

$$=\frac{\rho S}{2}V^3(1-a_1)[1-(1-2a_1)^2]=R(1-a_1)V.$$

This is, of course, the energy absorbed per unit time by the windmill.

If we now regard the windmill as moving with a velocity V into still air of pressure p, we have $b_1=2$ and $R=S\rho(1-a_1)2a_1V^2$, if as before

$(1-a_1)V$ represents the velocity of the air immediately in front of and measured relative to the windmill, and

$(1-a_1b_1)V$ represents the velocity relative to the windmill of the outflowing air across a section where the pressure is p.

In this case, however, the work done per unit time in moving the windmill through the air $=VR$. Also the kinetic energy of translation imparted to the air

$$=\rho S(1-a_1)\frac{V^3}{2}(2a_1)^2=a_1VR.$$

Hence the useful work done per unit time $=$(work done per unit time in moving the windmill through the air $-$energy imparted per unit time to the air of the outflowing stream)

$$=VR-a_1VR=VR(1-a_1).$$

Hence efficiency of working $=\dfrac{(1-a_1)VR}{VR}=(1-a_1)$.

We see, then, that when a windmill is rotating at a stationary point, the useful work is done at a sacrifice of the kinetic energy of the wind. When, however, a windmill is moving into still air, work has to be done against the resistance, a part of the work appearing as useful work and a part as kinetic energy of translation in the outflowing stream.

Theory of a Windmill Working in the Outflowing Stream from an Airscrew.—It is now proposed to consider very briefly the case of a windmill which, mounted on an aeroplane, works in the outflowing stream from the airscrew.

Let $V_1=$the velocity of the aeroplane, measured relative to the undisturbed air without,

and $V_2=$the velocity, measured relative to undisturbed air without, of the outflowing stream from the airscrew.

Then $V=V_1+V_2$, where V is the velocity of the windmill relative to the undisturbed air of the *outflowing stream*. If, then, $(1-a_1)V$ and $(1-a_1b_1)V$ represent the same velocities as before, we have $b_1=2$ and $R=\rho S(1-a_1)2a_1V^2$. Also work done on the windmill due to the forward velocity of the aeroplane $=RV_1$. As before, the energy—which eventually appears as useful work—absorbed by the windmill $=(1-a_1)VR$. Hence the ratio of the useful work done, to the work done on the windmill due to the motion of the aeroplane

$$=\frac{(1-a_1)VR}{R.V_1}=\left(1+\frac{V_2}{V_1}\right)(1-a_1).$$

Whenever practicable, then, it is advantageous, therefore, to mount a windmill in the outflowing stream from an airscrew.

An Aerofoil Theory of a Windmill.—It is convenient in the present case to consider the windmill to be moving with a velocity V into still air. In Fig. 120 is shown a sketch of a blade element, which is assumed to have an area Cdr, and to be at a radial distance r from the axis of rotation. In the present investigation it is considered that because of the resistance of the windmill the air in its immediate neighbourhood is moving forward with a velocity a_1V relative to undisturbed air.

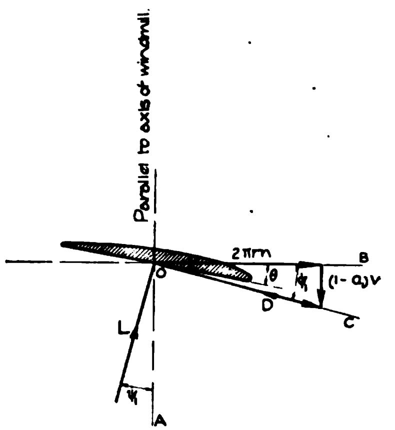


FIG. 120.

The blade element is therefore moving with a velocity $\sqrt{(1-a_1)^2V^2+(2\pi rn)^2}$ into the air in its immediate neighbourhood and is working at an angle of incidence of $(\psi_1-\theta)$, where $\tan\psi_1=\dfrac{(1-a_1)V}{2\pi rn}$. It should here be noticed that both θ and ψ are positive when measured in a clockwise direction. If k_L and k_D represent the absolute coefficients of lift and drag at this angle of incidence, it follows that the resistance of the blade element

$$=C.dr.[(1-a_1)^2V^2+(2\pi rn)^2][k_L\cos\psi_1+k_D\sin\psi_1]$$

and the useful torque developed

$$=C.dr.r[(1-a_1)^2V^2+(2\pi rn)^2][k_L\sin\psi_1-k_D\cos\psi_1].$$

Hence the resistance of the windmill

$$=\int_0^R N_BC.dr.[(1-a_1)^2V^2+(2\pi rn)^2][k_L\cos\psi_1+k_D\sin\psi_1]$$

and the useful torque developed

$$=\int_0^R N_B.C.dr.r[(1-a_1)^2V^2+(2\pi rn)^2][k_L\sin\psi_1-k_D\cos\psi_1],$$

where N_B = the number of blades.

The efficiency of working of the blade element

$$=\frac{2\pi rn[k_L\sin\psi_1-k_D\cos\psi_1]}{V[k_L\cos\psi_1+k_D\sin\psi_1]}=\frac{(1-a_1)\tan(\psi_1-\gamma)}{\tan\psi_1},$$

where $\tan\gamma=(k_D/k_L)$.

If a windmill be mounted on an aeroplane in the outflowing stream from the airscrew, it can easily be shown that the ratio of the useful work to the work done on the windmill by the aeroplane is $(1+V_2/V_1)\times(1-a_1)\,\dfrac{\tan(\psi_1-\gamma)}{\tan\psi_1}$ where V_1 is the velocity of the aeroplane relative to the undisturbed air and V_2 is the velocity of the outflowing stream from the airscrew measured relative to the undisturbed air.

In conclusion, it is apparent that the value of this aerofoil theory of a windmill will depend largely on the accuracy with which the value of a_1 is estimated.

Description of a Windmill Designed to Drive the Wireless Set of an Aeroplane.— A sketch of a windmill* designed at the National Physical Laboratory to drive the wireless set of an aeroplane is shown in Fig. 121. This windmill was mounted in the outflowing stream from the airscrew of an aeroplane which had a maximum horizontal flight speed of 85 m.p.h.; the corresponding velocity of the outflowing stream being approximately 96 m.p.h. At a

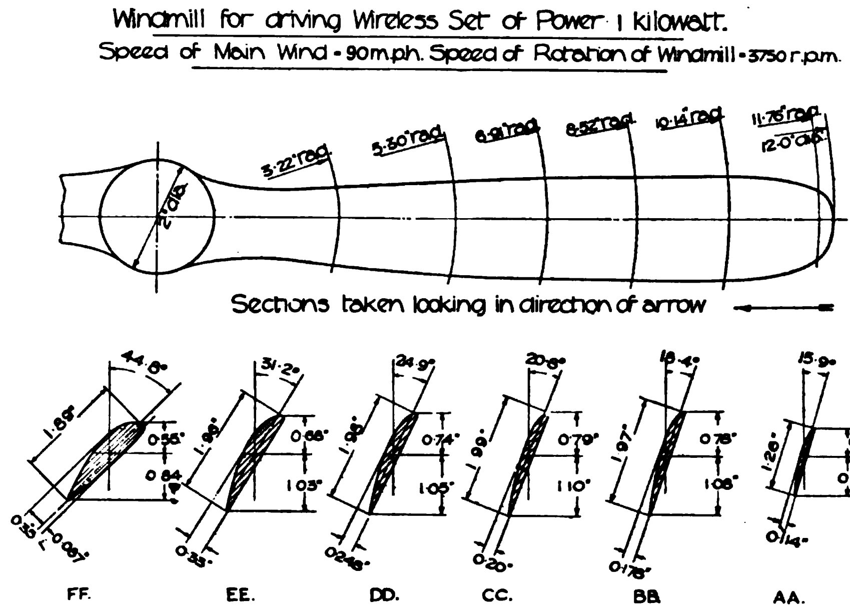


FIG. 121.

climbing speed of 63 m.p.h. the velocity of the outflowing stream would be approximately 92 m.p.h.

The generator of the wireless set gave, with an electrical efficiency of 64 per cent, a power of one kilowatt (1·34 h.p.) at a rotational speed of 3750 r.p.m. Accordingly, then, the windmill was designed to develop 1·34/·64, that is 2·1 h.p. at a rotational speed of 3750 r.p.m., the velocity of the windmill relative to the air in the outflowing stream from the airscrew being taken as 90 m.p.h. No allowance was made for the interference of the streamline casing of the generator with the performance of the windmill. From Fig. 121 it will be seen that the over-all diameter of the windmill is 2 ft., and that the angles of the blade vary from 15·9° at the tip to 44·8° at a section near the boss. The plan form is of rectangular shape, the ratio of the diameter to the maximum chord length being 12·0. When

* "A windmill to drive a wireless set of power one kilowatt," by A. Fage, A.R.C.SC., and H. E. Collins. Advis. Comm. Aeron., 1917.

transmitting messages, with the aeroplane flying at 85 m.p.h. the resistance of the windmill as calculated was 11·2 lb. Hence the ratio of the useful work to the work done on the windmill by the aeroplane

$$=\frac{2\cdot 1\times 60\times 550}{11\cdot 2\times 85\times 88}=0\cdot 83.$$

LIST OF SYMBOLS

V, Forward velocity of an airscrew relative to undisturbed air.
aV, Velocity of the inflowing air at the front of any blade element measured relative to the undisturbed air.
abV, Velocity of the outflowing air measured relative to the undisturbed air.*

In the case of an airscrew rotating at a stationary point ($V = 0$) the inflowing and outflowing speeds of the air are denoted by (aV_0) and (abV_0) respectively.

n, Rotational speed of an airscrew.
ω, Angular velocity of an airscrew.
D, Diameter of airscrew.
R, Radius of an airscrew.
r, Distance of a blade element from the axes of rotation.
C, Length of the chord of an aerofoil or a blade section.
σC, Length of the maximum ordinate of an aerofoil or a blade section.
S, Area.
S_B, Area of a blade section.
I, Moment of inertia of an airscrew about the axis of rotation.
I_B, Moment of inertia of a blade about the axis of rotation of the airscrew.
I_c, Moment of inertia of a blade section about an axis through the C.G. parallel to the chord.
I_N, Moment of inertia of a blade section about an axis through the C.G. at right angles to the chord.
D, Product of inertia about the axes through the C.G. of a blade section, which are parallel to and at right angles to the chord.
$\bar{Z}$, Height of the C.G. of a blade section above the chord.
L, Lift. The component of the force due to the relative wind which in the case of a blade element acts at right angles to the relative wind, and in the median plane of symmetry.
D, Drag. The component of the force due to the relative wind which in the case of a blade element acts along the relative wind and in the median plane of symmetry.
$\cot \gamma = \left(\frac{L}{D}\right)$, Aerodynamic efficiency of a blade section.
$p = \left(\frac{V}{n}\right)$, Effective pitch of an airscrew.
p_e, Experimental mean pitch of an airscrew.
$J = \left(\frac{V}{nD}\right)$, Pitch-diameter ratio of an airscrew.
g, Acceleration due to gravity.
t, Time.

* The Royal Aeronautical Society suggest that "bV" should represent velocity of the outflowing air, measured relative to the undisturbed air.

ρ, Absolute density of air.
Δ, Density of material of blade.
ν, Coefficient of kinematic viscosity.
E, Young's modulus of elasticity.
α, Angle of incidence of a blade section to the relative wind.
θ, Blade angle of a section, that is the angle between the chord of the section and a plane at right angles to the axis of rotation.
ψ, Angle between the direction of the relative wind at a blade section and a plane at right angles to the axis of rotation.
T, Thrust of an airscrew.
$\delta T = \delta r.t$, Thrust of a blade element.
Q, Torque of an airscrew.
$\delta Q = \delta r.q$, Torque on a blade element.
$k_L = \dfrac{L}{\rho.S.(Vely)^2}$, Absolute coefficient of lift.
$k_D = \dfrac{D}{\rho.S.(Vely)^2}$, Absolute coefficient of drag.
$k_T = \dfrac{T}{\rho n^2 D^4}$, Absolute coefficient of thrust.
$k'_T = \dfrac{T}{\rho V^2 D^2}$, Absolute coefficient of thrust.
$k_Q = \dfrac{Q}{\rho n^2 D^5}$, Absolute coefficient of torque.
$k'_Q = \dfrac{Q}{\rho V^2 D^3}$, Absolute coefficient of torque.
P, Power.
P_e, Effective Power = power absorbed by an airscrew.
P_u, Useful Power = power realised in the forward motion of an aeroplane.
η, Efficiency of an airscrew.
f, Frequency of vibration.
M, Mass of airscrew.

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